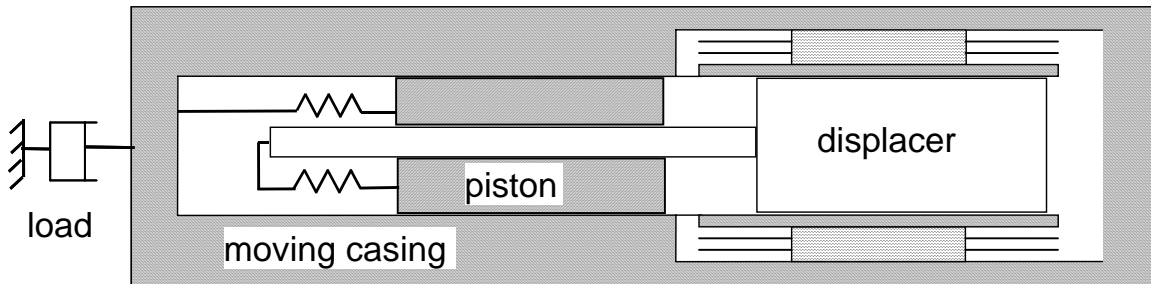


# Sage Model Notes

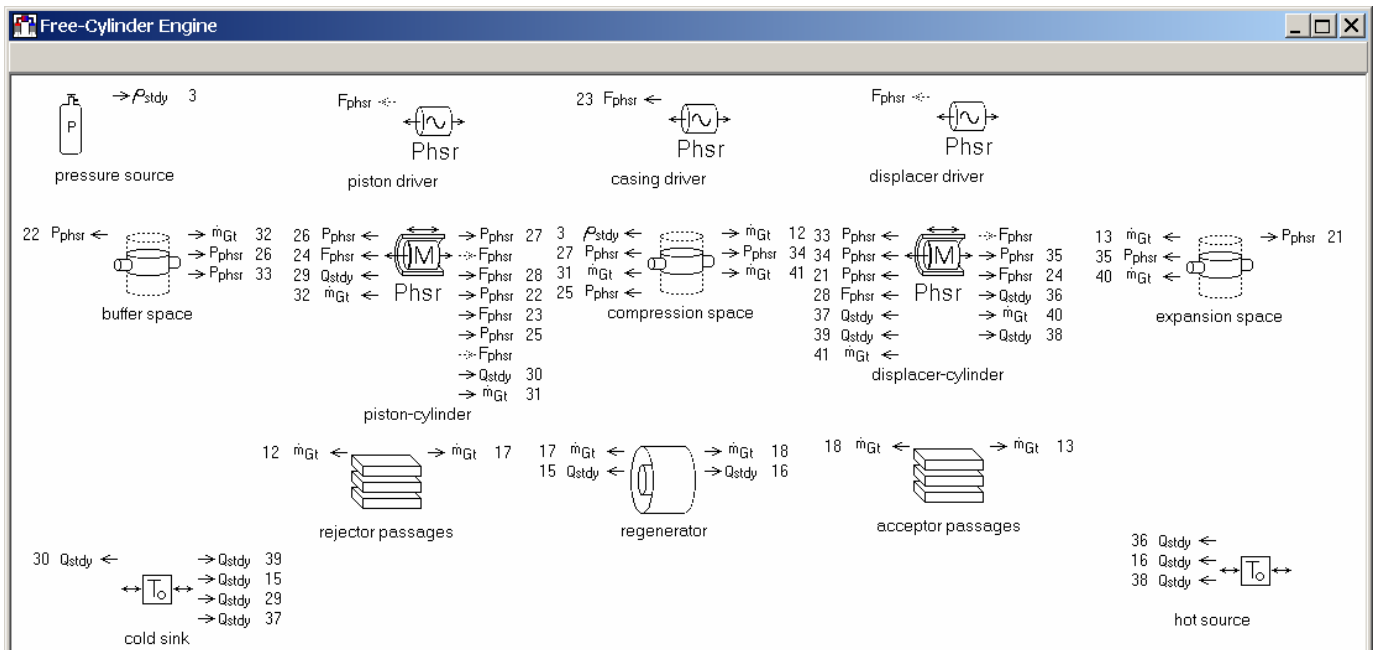
## FreeCyl.stl

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A model for a free-casing engine, sometimes called a free-cylinder engine. A free-casing engine is similar to a conventional free-piston engine except the piston is relatively heavy compared to the pressure enclosure or *casing* which is not anchored down. As a result the casing motion (relative to a fixed inertial reference frame) is large and is used to deliver power to an external load. Here is a rough schematic of the physical layout:

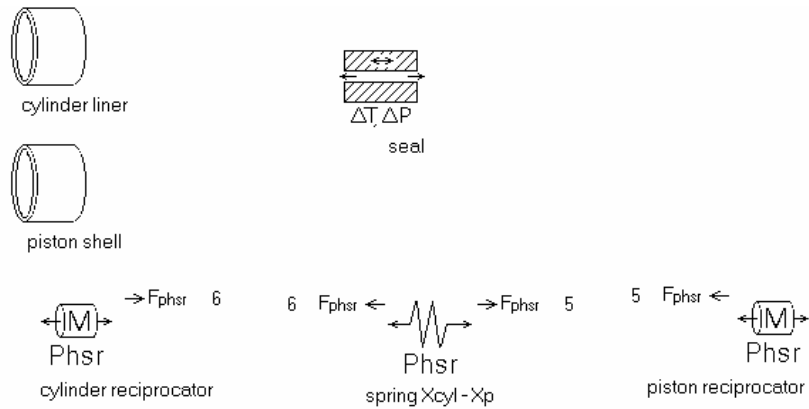


The Sage model looks like this:



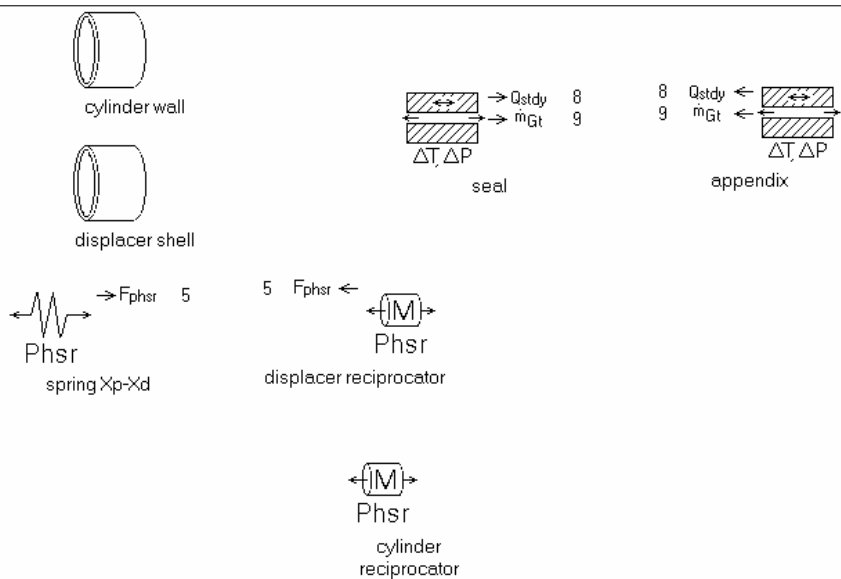
In the model the casing is not actually free. The *casing driver* component forces it to move at a specified amplitude and phase ( $Xamp$  and  $Xphase$  inputs) and the load is represented only in terms of the forcing function  $FF$  required to absorb the total mechanical power delivered to the casing, eventually the result of the PV power delivered by the working gas to the compression and expansion space boundaries.  $FF$  is an output variable of the *cylinder driver* component.

Within the Sage model the masses and dynamic interactions of the three moving components are represented inside the *piston-cylinder* and *displacer-cylinder* components. Here is what is inside the the *piston-cylinder* component:



The *cylinder liner* and *piston shell* are built-in components normally used to represent thermal conduction but not important in this model because there is no temperature gradient in either. The *seal* represents gas flow between the compression and buffer spaces through a clearance seal. The *cylinder reciprocator* component represents part of the casing mass and the *piston reciprocator* component represents the piston mass. A relative spring connects the two. Both reciprocators have force attachments attached to the top-level *driver* components and frontal area attachments to the *compression space* (between piston and displacer in above schematic), *expansion space* (between displacer and right interior casing wall) and *buffer space* (between piston and left interior casing wall). There are more area attachments used in a free-casing model than a fixed-casing model because areas attached to the casing are not stationary and must be considered when calculating volume displacements. For example, if the piston and displacer diameters were not equal then the motion of the casing (with piston and displacer fixed) would produce a volume displacement in the compression space. In the model, the cylinder reciprocator has an area attachment (*cyl cs-facing area*) representing this displacement. It is currently set to zero ( $1.0E-10 \text{ m}^2$ , actually) consistent with the two cylinder diameters being equal (as represented in the schematic).

Here is what is inside the *displacer-cylinder* component:



This time the *cylinder wall* and *displacer shell* are meaningful because they represent the solid material and wall thicknesses of the two walls and there is a temperature gradient. The conductive surface within the appendix represents the combined wall conduction. The *seal* and *appendix* represents gas flow between compression and expansion spaces in an annular clearance seal followed by a larger annular gap along the part of the displacer with the temperature gradient. The *cylinder reciprocator* represents the remainder of the moving casing mass. It is connected by a force connection to the component by the same name in the *piston-cylinder* component so the two move with the same amplitude and phase. It is connected with a relative spring to the *displacer reciprocator* as a means to achieve the desired phase relationships of the moving components. Within the displacer reciprocator the area attachment *dis bs-facing area* represents the drive rod area. And so forth.

## Dynamic Analysis

There is an optimization specification which solves for the implied operating frequency based on the constraint that the load is a pure damper. In other words the load contains no "spring" or "mass" elements. This constraint is implemented within the *casing driver* as  $FF.Real = 0$  which means in English that the real part of the required forcing function is zero. Since the *casing driver* phase is set to 180 degrees (so the piston phase will be close to the customary zero degrees), the real part of  $FF$  is the part in phase with the motion. The part produced by a spring or mass attachment. A pure damper produces a force in phase with velocity, corresponding to the imaginary part of  $FF$  ( $FF.Imag$ ).

One thing you can do with this optimization is to change the cylinder amplitude and see how frequency and power output change as a result. The assumption is that the load device always adjusts to absorb the power output by supplying just the required amount of pure damping force. A real load is not like this, of course and the actual load behavior is important for producing stable operation. The power absorbed by the load must grow faster as a function of cylinder amplitude than the power produced by the engine or the operating point will be unstable. In the current model state the piston and displacer components are free, meaning that their amplitudes and phases are determined by the gas and spring forces acting upon them. As an alternative, the *piston driver* and *displacer driver* components may be used to constrain the piston and displacer to move as you want. To do this simply connect the force attachments to the corresponding free components (reciprocators) using the connector arrows already in the model. This puts the model in *constrained* piston and displacer mode, which is handy for design purposes.

When the model is in constrained mode there may be external forces required to produce the piston and displacer motions as indicated by the forcing function output  $FF$  for the piston driver and displacer driver. You can use the optimizer to adjust various inputs in order to zero these forces, which means that the piston and displacer equations of motion are balanced and the motions will not change if the connections to the piston and displacer driver are removed. This sort of thing is an essential part in the design of any free-piston machine. For example you might optimize the displacer rod area (area attachment named *dis bs facing area* within the displacer) and displacer spring stiffness (named *spring Xp-Xd*) in order to make  $FF.Real = 0$  and  $FF.Imag = 0$  simultaneously in the displacer driver. For the piston you might optimize the mass and piston phase angle subject to the same constraints in the piston driver. This could be done in conjunction with the optimization of a number of other variables as well with the objective of maximizing power output for a given heat input, or whatever. The result would be a free-cylinder engine design that should run at the design point (if built corresponding to the model), although maybe not stably, depending on the load characteristics.

## Bottom-Line Outputs

Net heat input, heat rejection, PV power and indicated efficiency are available in the root model as the following user-defined variables:

Qin	net heat input	1.314E+03
	Qacceptor + Qhotsource	
Qrej	Net heat rejection	-1.031E+03
	Qrejector + Qcoldsink + QpisSeal + QdisSeal + Qbuff	
Wpv	total pv power	2.836E+02
	PVes + PVcs + PVbs	
Eff	Efficiency	2.158E-01
	Wpv/Qin	

Also available in the root model are a number of user-defined variables that represent the phasor components of motions and the various forces acting on the moving parts. These are:

XpRelReal		1.240E-02
	XpReal - XcReal	
XpRelImag		-1.725E-03
	Xplmag - Xclmag	
XpRelAmp	amp Xp - Xc	1.252E-02
	Sqrt( Sqr(XpRelReal) + Sqr(XpRelImag) )	
XpRelPhase	phase Xp - Xc	-7.921E+00
	57.30 * ArcTan( XpRelImag / XpRelReal )	
XdRelReal		-2.153E-03
	XdReal - XcReal	
XdRelImag		4.913E-03
	Xdlmag - Xclmag	
XdRelAmp	amp Xd - Xc	5.364E-03
	Sqrt( Sqr(XdRelReal) + Sqr(XdRelImag) )	
XdRelArg	phase Xd-Xc	-6.634E+01
	57.30 * ArcTan( XdRelImag / XdRelReal )	
DPdisAmp	DP across displacer	3.070E+04
	Sqrt( Sqr(PesReal-PcsReal) + Sqr(PesImag-PesImag) )	
DPdisPhase		5.587E+01
	57.30 * ArcTan( (PesImag-PcsImag) / (PesReal-PcsReal) )	
FrodReal	disp rod force	-4.689E+02
	Arod * (PbsReal-PesReal)	
FrodImag	disp rod force	1.009E+02
	Arod * (PbsImag-PesImag)	
FdpReal	displ drag force	4.667E+01
	AdisNeg * (PcsReal-PesReal)	
Fdplmag	disp drag force	6.885E+01
	AdisNeg * (PcsImag-PesImag)	

All of the above are calculated from lower-level user-defined variables that have been exported to the root level for referencing purposes.