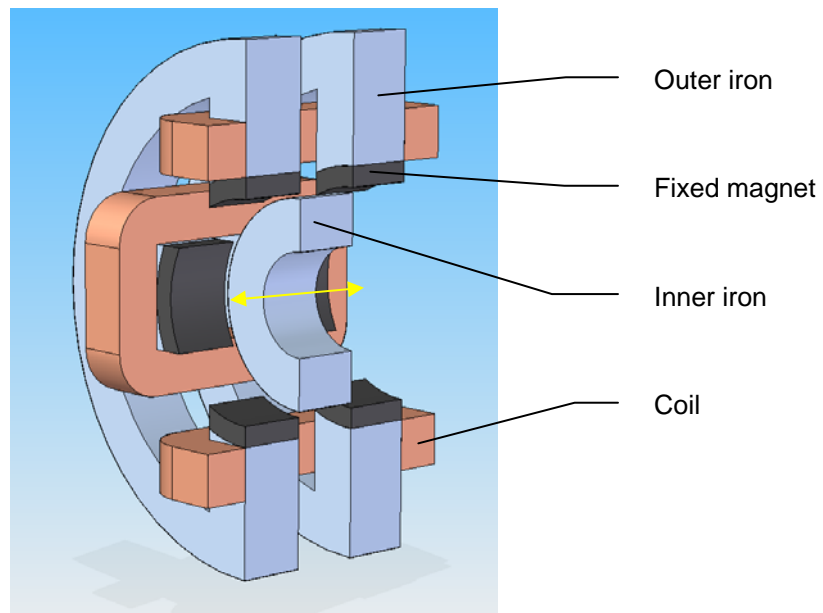


Sage Model Notes

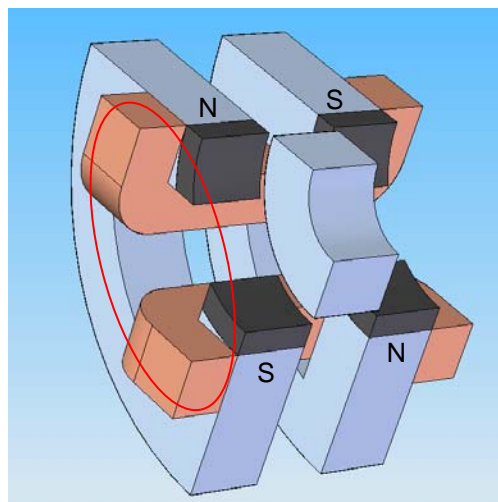
MotorMovIron.stl

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A model of moving iron type linear motor (alternator) consisting of a moving inner iron piece and two fixed outer iron magnetic flux paths, each with four arms. Permanent magnets are attached at the ends of the arms and a coil is wrapped around each adjacent pair of arms, as shown here in a section view:



The magnetic flux path is a bit confusing. In this motor there are actually four equivalent magnetic circuit quadrants, one of which is depicted in this quarter-section view:

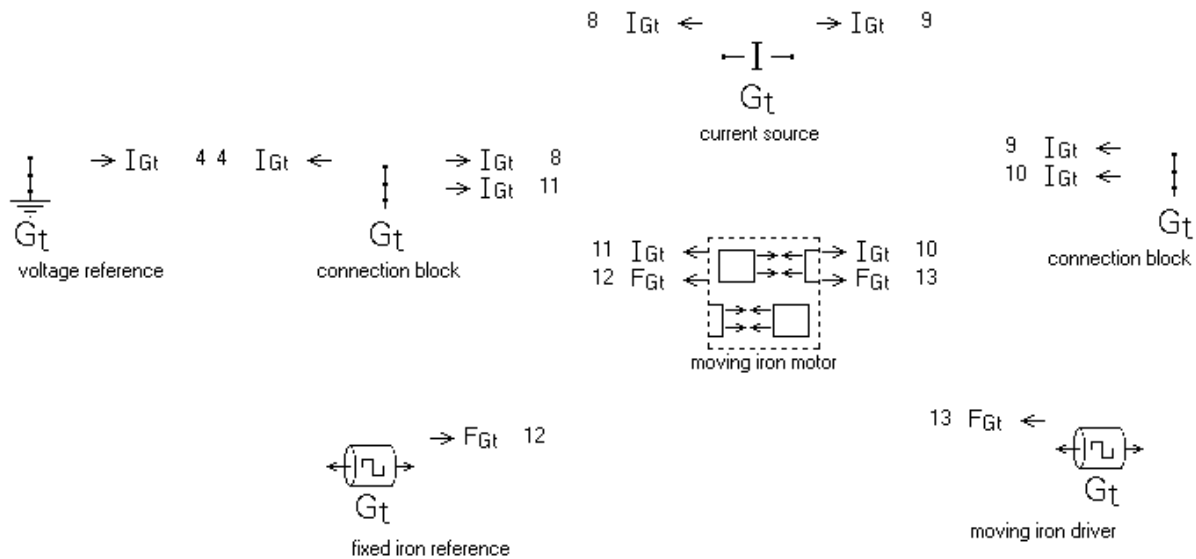


The red oval indicates the general magnetic flux direction, although the flux is within the iron instead of outside as drawn. The magnitude and sign of magnetic flux varies with the

position of the inner iron piece. For the central position shown there is no net flux. At the extreme right or left inner iron positions the flux linked through the coils is maximum in opposite directions. The structure that actually moves and aligns the inner pole piece is not shown.

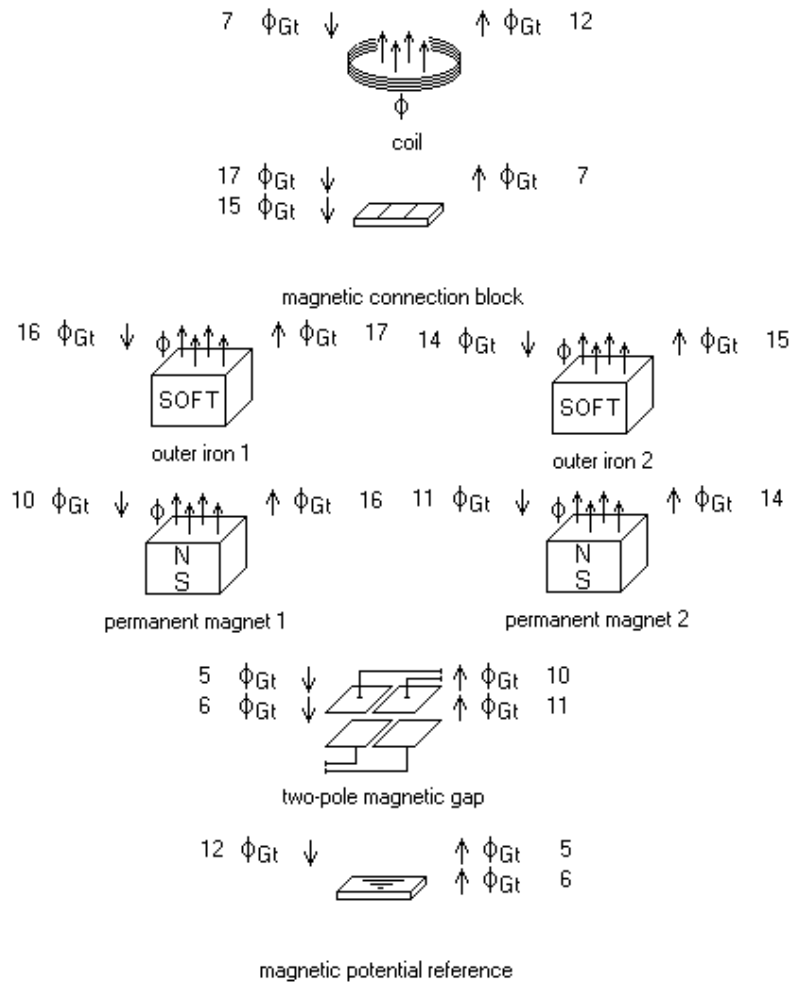
Beware Side Forces In this design the side forces resulting from axial mis-alignment are potentially greater than for the moving magnet design (*MotorMovMag.stl*). In the quadrant view above, if the inner iron core is displaced radially in the direction midway between the two arms it will reduce the air gap and substantially increase the magnetic flux in that quadrant, while doing the opposite in the quadrant diametrically opposite. This will produce a side force that grows with radial displacement. The root of the problem is the magnetic flux path turning circumferentially in the moving iron. For the concentric moving magnet design (*MotorMovMag.stl*) there is no such problem because the flux path is always radially directed in the moving piece (magnet) so the total air gap along a typical flux loop does not vary with radial displacement. Sage does not model side forces so the issue of side forces and axial alignment must be addressed by separate magnetic and structural analysis.

The Sage model looks like this:



A *current source* (top row) drives electrical current through the coil within the *moving iron motor* submodel. A constrained piston *fixed iron reference* anchors the motor outer iron assembly and another constrained piston *moving iron driver* drives the inner iron, receiving mechanical power. In this model the phase of the current is set 90 degrees ahead of the phase of the magnet motion. Both are independent inputs. The current phase difference determines whether the model corresponds to a motor (mechanical power producer) or alternator (mechanical power absorber) or something in between (some component of magnetic force in phase with the motion like a spring).

Within the *moving magnet motor* submodel are these components:



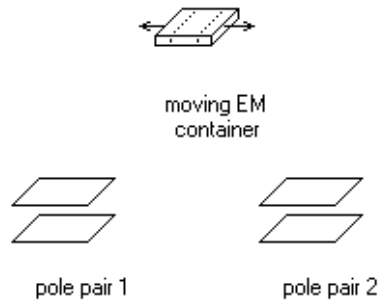
The actual multiple-arm outer geometry is folded into equivalent *outer iron* and *coil* components. Effectively the outer iron path in the above quadrant view is bundled into N parallel paths represented by dual iron and magnet paths of the same length and total cross-section area. The N coils are effectively wired in series into a single coil.

To simplify the model the magnets on the ends of the arms in the quadrant view are replaced by two double-thickness magnets on a single arm. The magnets are polarized in opposite directions by setting the polarization multiplier input *Jmult* to -1 in the component *permanent magnet object 1*.

In the *two-pole magnetic gap* the lower poles of pole pairs 1 and 2 are anchored to magnetic potential references with the same zero potential as the upper pole of the *coil* magnetic path. The upper poles of pole pairs 1 and 2 are connected to the lower poles of the two *permanent magnets*, then in series with the two *outer iron* paths, then combined into a single flux path passing through the *coil* using the *magnetic connection block*.

The Sage model captures the magnetic potential drops across the *two-pole magnetic gap* correctly but puts the equi-potential surface at the lower pole faces rather than the air gap mid-point. The Sage model is not quite physically correct but should produce a reasonable approximation of the magnetic flux in the two halves of the magnetic gap.

Inside the *two-pole magnetic gap* are the *moving EM container* in which the moving iron resides:



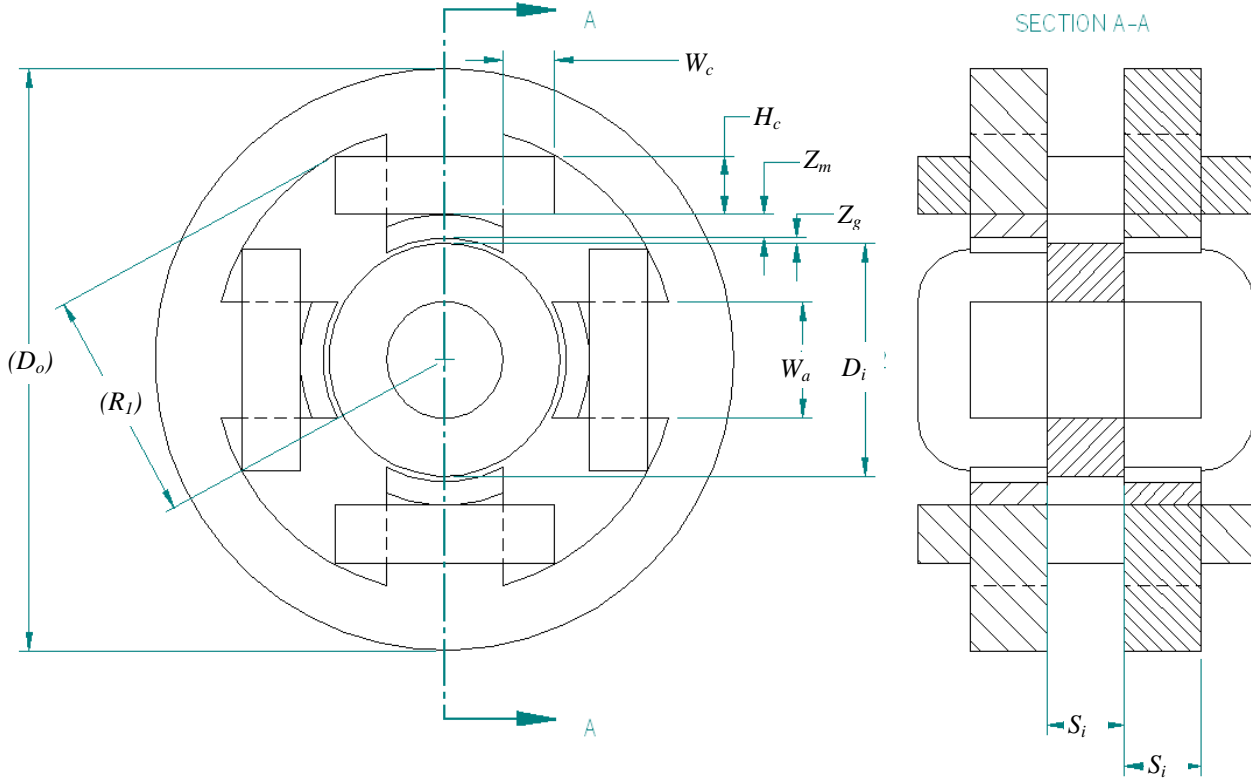
The idea is that the iron inside the moving EM container passes between pole pairs 1 and 2 and drives flux through the external magnetic path accordingly.

Beware 1-D assumptions Sage assumes the moving iron magnetic flux is always directed in a plane normal to the motor axis (Sage z direction). This assumption is reasonable when the moving iron is aligned with the outer poles at the extreme ends of its stroke or is between the poles at mid stroke. But when there is only a partial overlap between the poles and moving iron there will be some flux spreading in the axial direction. The Sage model will not capture this flux spreading so will tend to under-predict the magnetic flux for a given magnetic potential difference.

Sage also assumes the magnetic potential is uniform across the pole faces, which in this design are the magnet faces. Uniform pole potential is reasonable when the poles are made of an isotropic highly permeable ferromagnetic material (soft iron) because the variation of magnetic potential will be relatively low in any direction. But for a permanent magnet the magnetization is locked into the material and the face of the magnet overlapping the moving iron piece will be at a different potential than the face of the magnet over the air gap.

For these reasons a Sage model of a moving iron motor may not be highly accurate and should be backed up by multi-dimensional magnetic analysis.

There are user-defined inputs defined in the *moving iron motor* submodel based on the symbols in the dimensioned picture below. The symbols in parenthesis are dependent values calculated from the independent values without parenthesis. In the Sage model the symbols in parenthesis correspond to user-defined variables.



The moving inner iron length S_i is the same as the spacing between outer poles with an available motion amplitude of $\pm S_i$ from the center position as drawn before it moves beyond the ends of the magnets. For this model the design-point moving iron amplitude is also S_i .

From the Pythagorean theorem the outer iron inner radius is:

$$R_1 = \sqrt{\left(D_i/2 + Z_g + Z_m + H_c\right)^2 + \left(W_a/2 + W_c\right)^2}$$

The magnetic flux in the radial arms splits into two directions in the outer ring so the radial width of the outer ring that produces equal flux-path area is $W_a/2$. The outer iron diameter is therefore

$$D_o = 2\left(R_1 + W_a/2\right)$$

The outer-iron mean flux path length (single quadrant) is roughly the sum of the radial distances up and down two arms plus the circumferential arc length, or

$$L_o = D_o - \left(D_i + 2Z_g + 2Z_m + W_a/2\right) + \pi\left(D_o - W_a/2\right)/N$$

Where N is the number of arms. The inner iron flux path length is by similar calculation roughly

$$Z_i = W_a/2 + \pi(D_i - W_a/2)/N$$

In the Sage model Z_i is the z-directed iron thickness within the magnetic gap.

The coil cross section area for a single coil is:

$$A_c = W_c H_c$$

To calculate the coil centroid diameter ($D_c = A_c/(\pi V_c)$) requires the coil volume. By breaking the coil into the sum of rectangular and corner pieces the volume of a single coil is

$$V_c = 2A_c(2S_i + W_a) + H_c \pi W_c^2$$

The model combines all the radial arms into a single equivalent magnetic path and all the coils into a single equivalent coil, as established by these user defined inputs and outputs in the *moving iron motor* component:

Inputs

Narm	number radial arms (NonDim)	4.000E+00
Si	moving iron length (m)	2.000E-02
Di	OD moving iron (m)	4.000E-02
Wa	radial arm width (m)	2.000E-02
Zm	magnet thickness (m)	4.000E-03
Zg	air gap (m)	1.000E-03
Sm	magnet axial separation (m)	2.000E-03
Hc	coil height (m)	1.000E-02
Wc	coil width (m)	9.000E-03

Outputs

R1	outer iron inner radius	3.982E-02
	$\text{Sqrt}(\text{Sqr}(0.5*Di + Zm + Zg + Hc) + \text{Sqr}(0.5*Wa + Wc))$	
Dout	outer iron OD	9.965E-02
	$2*(R1 + 0.5*Wa)$	
Lout	mean outer iron path length	1.101E-01
	$Dout - (Di + 2*Zg + 2*Zm + 0.5*Wa) + \text{Pi}*(Dout - 0.5*Wa)/Narm$	
Zi	moving iron mean path length	3.356E-02
	$0.5*Wa + \text{Pi}*(Di - 0.5*Wa)/Narm$	
Ac	coil cross section	3.600E-04
	$Narm*Wc*Hc$	
Vc	coil volume	1.830E-04
	$Narm*(Ac*2*(2*Si + Wa) + Hc*\text{Pi}*\text{Sqr}(Wc))$	

Per the above geometry the model component inputs are recast to these values:

Outer Iron recasts

$$L_{\text{path}} = L_{\text{out}}$$

$$A_{\text{path}} = N_{\text{arm}} * 2 * S_i * W_a$$

Permanent magnet recasts

$$L_{\text{path}} = 2 \cdot Z_m$$

$$A_{\text{path}} = N_{\text{arm}} \cdot S_i \cdot W_a$$

Permanent magnet object recasts

$$\text{ThkLam} = W_a$$

Two-pole magnetic gap recasts

$$Z_{\text{gap}} = Z_i + 2 \cdot Z_g$$

$$W_{\text{pole}} = N_{\text{arm}} \cdot 2 \cdot S_i$$

$$L_{\text{pole1}} = S_i - 0.5 \cdot S_m$$

$$X_{\text{gap}} = S_m$$

$$L_{\text{pole2}} = S_i - 0.5 \cdot S_m$$

Coil recasts

$$D_{\text{centroid}} = V_c / (\pi \cdot A_c)$$

$$D_{\text{wire}} = \sqrt{4 / \pi \cdot A_w}$$

A_w is the user defined variable

A_w	wire section	6.480E-07
$\text{Alpha} \cdot A_c / N_{\text{turns}}$		

The wire diameter is recast so that the coil fits into the overall cross-section area. The total coil cross section area is A_c , which establishes the cross section area of an individual wire as

$$A_w = \frac{\alpha A_c}{N}$$

Where α is the coil packing factor and N is the number of turns. The wire diameter must then be

$$D_w = \sqrt{\frac{4}{\pi} A_w}$$

Moving EM container recasts

$$\text{Length} = S_i$$

$$\text{Offset} = S_i$$

The length (inner iron length) and offset are based on the assumption that the inner iron endpoints will coincide with the pole endpoints at the extremes of its stroke S_i .

Moving iron material recasts

$$Z_{\text{thkRel}} = Z_i / (Z_i + 2 \cdot Z_g)$$

Energy Balance

It is helpful to consider the energy balance in the stationary parts separate from the moving iron. The following table accounts for the energy flows from the current source to the magnetic energy flowing into the gaps between the poles of the *two-pole magnetic gap*.

	Power W
Input power from current source (Fwe)	-1.620E+02
Coil I^2R loss (Wdissip)	3.376E+00
Outer iron 1 eddy-current loss (Weddy)	2.693E-01
Outer iron 1 hysteresis loss (Whyst)	8.929E-02
Outer iron 2 eddy-current loss (Weddy)	1.638E-01
Outer iron 2 hysteresis loss (Whyst)	8.612E-02
Magnet 1 eddy-current loss (Weddy)	1.074E+01
Magnet 2 eddy-current loss (Weddy)	6.532E+00
Net power into magnetic gap	-140.7

So there is 140.7 W magnetic power flowing into the magnetic gap that is potentially available to do mechanical work. This same power should also be the sum of the mean values for the Fw_m (magnetic power inflow) outputs for components pole pair 1 and pole pair 2, which is 140.7 W, in good agreement.

The next table shows where the magnetic gap incoming power goes:

	Power in W
Moving iron eddy-current loss (Weddy)	1.579E+00
Moving iron hysteresis loss (Whyst)	2.274E-01
Mechanical power output (-W of moving EM container)	1.388E+02
Total	140.6

So energy is conserved within round-off error.

The predicted motor efficiency is **0.857** (138.8 / 162.0) with the largest losses being coil resistance loss and magnet eddy current losses.