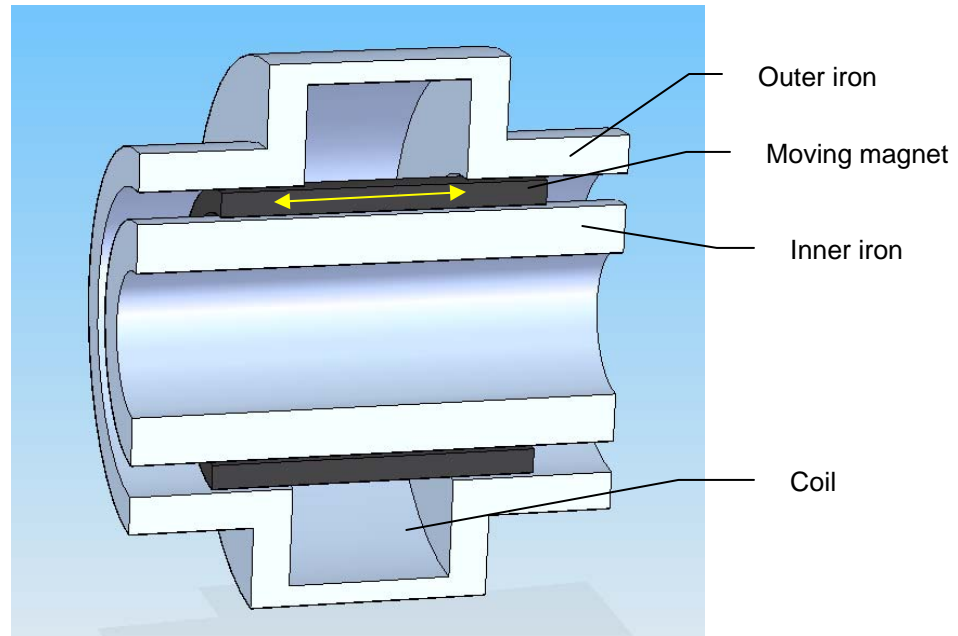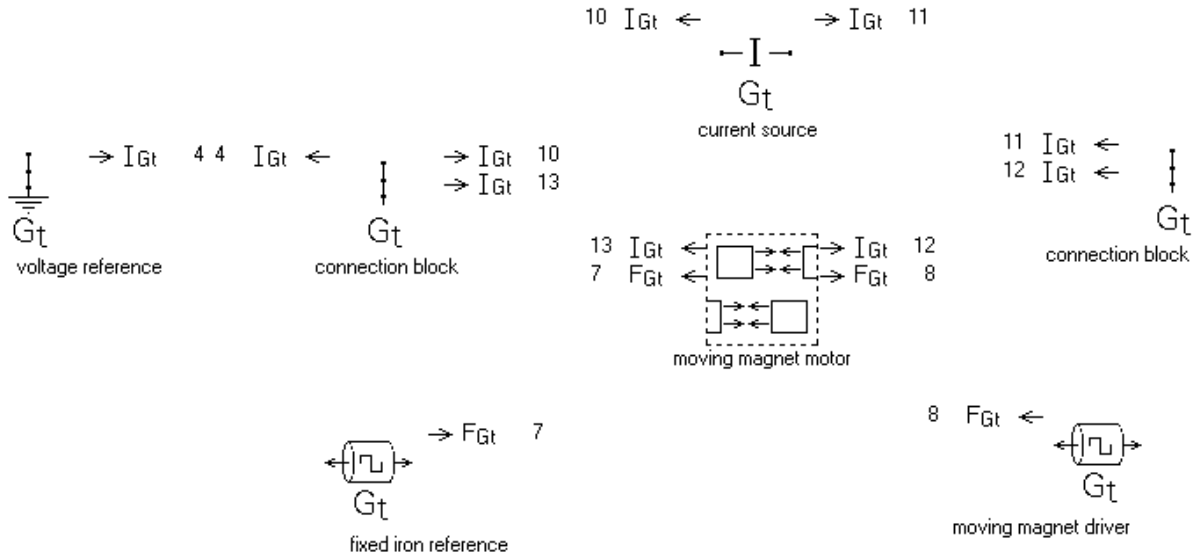*Sage Model Notes*

MotorMovMag.stl

D. Gedeon
4 June 2012

A model of moving magnet type linear motor (alternator) consisting of a magnet ring moving between inner and outer iron magnetic flux paths, shown here in a section view:

The coil winds circumferentially in the space indicated. The magnet is radially polarized and moves axially back and forth in the direction of the arrows driving magnetic flux alternately up and down (radially) within the gaps between inner and outer iron pieces. This results in an alternating axial magnetic flux in the inner iron, which is the flux linked through the coil. The flux completes the magnetic circuit via the outer iron path. The structure that actually moves and aligns the magnet is not shown.
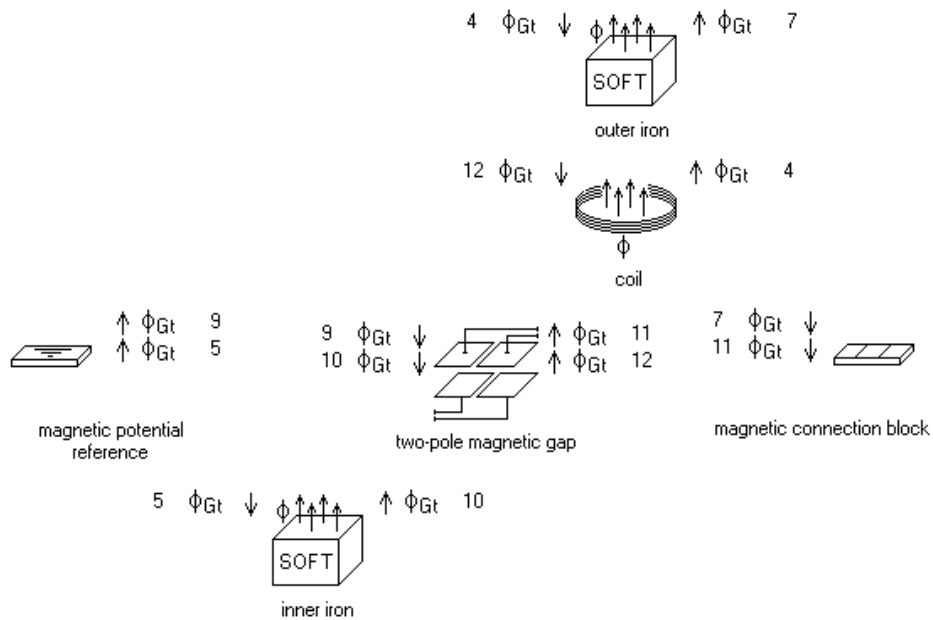
The Sage model looks like this:

A *current source* (top row) drives electrical current through the motor coil within the *moving magnet motor* submodel. A constrained piston *fixed iron reference* anchors the motor pole pieces and another constrained piston *moving magnet driver* drives the moving magnet, receiving mechanical power. In this model the phase of the current is set 90 degrees ahead of the phase of the magnet motion. Both are independent inputs. The current phase difference determines whether the model corresponds to a motor (mechanical power producer) or alternator (mechanical power absorber) or something in between (some component of magnetic force in phase with the motion like a spring).
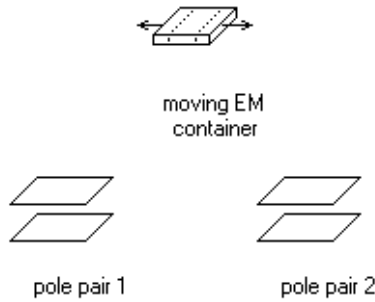
If this were an alternator model the current source might be replaced by a load resistor. Except, due to inductance, voltage and current are about 90 degrees out of phase in the coil so there must also be a tuning capacitor in series with the resistor in order to cancel the inductive component of the coil voltage so that voltage and current can be in phase in the resistor.

Within the *moving magnet motor* submodel are these components:

The idea is that the magnet inside the moving EM container passes between pole pairs 1 and 2 and drives flux through the external magnetic path accordingly.
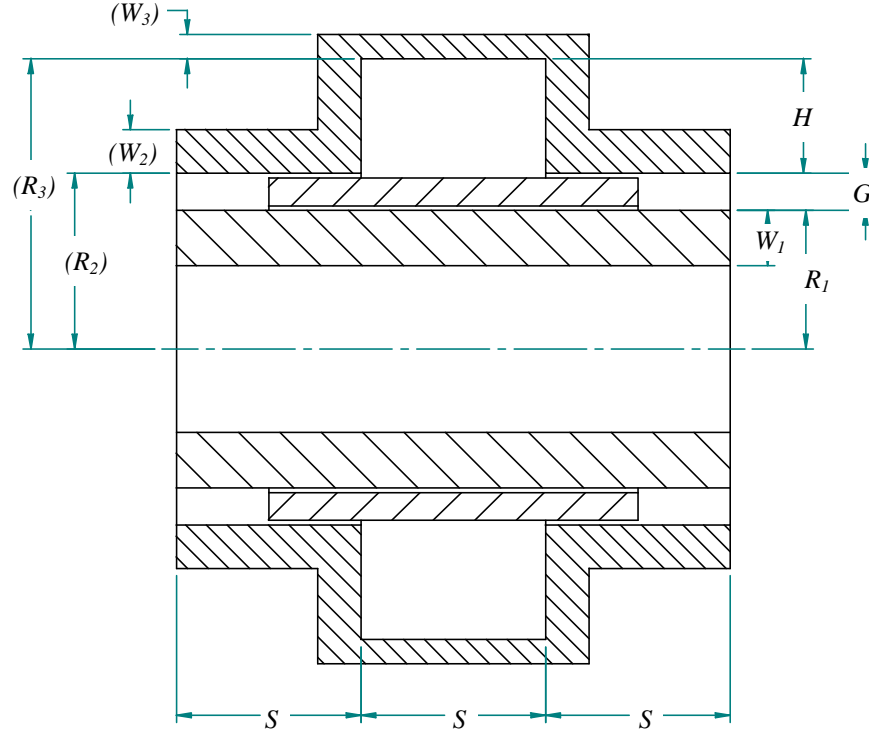
Admittedly the Sage motor submodel looks nothing like the actual motor. The inner and outer iron pieces are represented by two linear paths connected in series with each other and the coil across the two pole pairs of the *two-pole magnetic gap*. Tracing connection arrows shows this is so.

Inside the *two-pole magnetic gap* is the *moving EM container* in which the moving magnet resides:

The idea is that the magnet inside the moving EM container passes between pole pairs 1 and 2 and drives flux through the external magnetic path accordingly.

There are user-defined inputs defined in the *moving magnet motor* submodel based on the symbols in the dimensioned picture below. The symbols in parenthesis are dependent values calculated from the independent values without parenthesis. In the Sage model the symbols in parenthesis correspond to user-defined variables.

For simplicity the pole pieces and coil space all have the same length S. In this model S is twice the design point magnet amplitude. The magnet is drawn at its center position. So at its extreme negative (left) position the magnet fully fills the gap of pole pair 1 and likewise for the gap of pole pair 2 at the extreme positive position.

The independent variables are on the right in terms of which the dependent variables on the left are:

$$R_2 = R_1 + G$$
$$R_3 = R_2 + H$$
$$W_2 = W_1 \frac{R_1}{R_2}$$
$$W_3 = W_1 \frac{R_1}{R_3}$$

The W values make the cross section areas of all the iron paths approximately the same. Because the product WR = constant for all. Setting the value W_1 sets them all. The total coil cross section area is:

$$A_c = SH$$

In the model the above variables are defined in the *moving magnet motor* component as:

Inputs

Spole	pole length (m)	2.000E-02
R1	inner iron radius (m)	1.500E-02
Gmag	magnet gap (m)	3.000E-03
Hcoil	coil height (m)	1.200E-02
W1	inner iron thickness (m)	6.000E-03

Outputs

R2	outer iron radius	1.800E-02
R1 + Gmag		
R3	coil outer radius	3.000E-02
R2 + Hcoil		
W2	outer iron inner thickness	5.000E-03
W1 * R1 / R2		
W3	outer iron outer thickness	3.000E-03
W1 * R1 / R3		
Ac	coil cross section	2.400E-04
Spole * Hcoil		

Both flux path cross section areas are Apath = $2\pi RW$, independent of subscripts. The mean flux path length in the inner iron is roughly Lpath = 2 S (mid pole to mid pole). For the outer iron it is roughly Lpath = 2 $(S + H)$, which includes the radial segments along the coil height. The inputs for the inner and outer iron component are recast to these values:

Inner Iron recasts
Lpath = 2 * Spole
Apath = 2 * Pi * R1 * W1

Outer Iron recasts
Lpath = 2*(Spole + Hcoil)
Apath = 2 * Pi * R2 * W2

The coil Dcentroid input is recast to the mean coil diameter 2 R_2 + H.

The wire diameter is recast so that the coil fits into the space in the outer iron piece. The total coil cross section area must be $Ac = SH$, which establishes the cross section area of an individual wire as

$$A_w = \frac{\alpha A_{cutout}}{N}$$

Where α is the coil packing factor and N is the number of turns. The wire diameter must then be

$$D_w = \sqrt{\frac{4}{\pi} A_w}$$

Coil recasts
Dcentroid = 2 * R2 + Hcoil
Dwire = Sqrt(4/Pi * Aw)

In the two-pole magnetic gap component the width is just the magnet circumference

$$W = 2\pi\left(R_1 + \tfrac{1}{2}G\right)$$

And other inputs come directly from the drawing:

Two-pole magnetic gap recasts
  Zgap = Gmag
  Wpole = 2*Pi * (R1 + 0.5*Gmag)
  Lpole1 = Spole
  Xgap = Spole
  Lpole2 = Spole

For the *moving EM container* component the length (magnet length) and offset are based on the assumption that the magnet endpoints will coincide with the pole endpoints at the extremes of its stroke. For magnet stroke (twice amplitude) equal to pole length *S* this works out to:

Moving EM container recasts
  Length = 2*Spole
  Offset = 0.5*Spole

The thickness of the permanent magnet is presumed to be only 80% the thickness of the total air gap, which is set by this input of the *permanent magnet* component:

  ZthkRel   thickness fraction of parent Zgap [0, 1] 8.000E-01

Energy Balance

The energy flows in this model are somewhat more complicated than for a stationary component model --- and more interesting.

It is helpful to consider the energy balance in the stationary parts separate from the moving magnet. The following table accounts for the energy flows from the current source to the magnetic energy flowing into the gaps between the poles of the *two-pole magnetic gap*.

	Power W
Input power from current source (Fwe)	-1.745E+02
Coil I^2R loss (Wdissip)	1.503E+00
Inner iron eddy-current loss (Weddy)	7.813E-01
Inner iron hysteresis loss (Whyst)	1.869E-03
Outer iron eddy-current loss (Weddy)	1.250E+00
Outer iron hysteresis loss (Whyst)	2.990E-03
Net power into magnetic gap	**-171.0**

So there is 171.0 W magnetic power flowing into the magnetic gap that is potentially available to do mechanical work. This same power is also the sum of the mean values for the FWm (magnetic power inflow) outputs for components pole pair 1 and pole pair 2, which are calculated (per unit volume) as

$$\frac{dE_m}{dt} = H\frac{dB}{dt}$$

The next table shows where the magnetic gap incoming power goes:

	Power in W
Magnet energy loss (FWm of permanent magnet embedment)	7.723E+00
Mechanical power output (-W of moving EM container)	1.633E+02
Total	**171.0**

The 7.7 W magnetic energy loss is the magnetic power inflow to the magnet in the moving magnet reference frame. Of that, 2.67 W is due to eddy current losses (output Weddy of permanent magnet embedment) based on the effective lamination thickness input.

ThkLam   lamination thickness (m)   1.000E-02

The remaining 5.0 W loss is equivalent to a magnet hysteresis loss except there should be no hysteresis loss for a permanent magnet. The magnetization is taken as a single-valued function of magnetic flux. What is happening is that numerical errors in Sage's coarse time grid give rise to an *effective* hysteresis loss because the steep change in the magnetic flux distribution as the magnet passes through the pole boundaries are difficult to resolve in the time domain. The net result is that Sage is somewhat conservative in predicting motor efficiency.

The predicted motor efficiency is **0.936** (163.3 / 174.5). Without the numerical hysteresis the mechanical output would be 5.0 W higher and the motor efficiency would be **0.965**.

Mechanical power output

Normally the principle of energy conservation for energy flows between components in a Sage model is guaranteed by the numerical differencing formulas. But this is not the case for the mechanical power output which results from a calculation of magnetic force on a component moving through an external magnetic field. There can be an energy leak (hopefully small) in the solution!

But global energy conservation is too important to abandon. Without energy conservation the optimizer might take advantages of any errors favorable to motor efficiency, or whatever. So Sage calculates Mechanical power output so as to always preserve at least the time average global energy conservation. The moving EM container solution contains a slack variable:

Fscale   magnetic force correction (NonDim)   9.758E-01

This variable scales the value of the magnet magnetic force calculation and is adjusted as part of the solution so that the time-average power output conserves energy relative to the magnetic energy flow into the pole gaps. The fact that the value is very close to one confirms that the energy leak of the solution is relatively small.

Solution Grid

You can understand more about the model by dumping the solution grid for the two-pole magnetic gap component, which contains the grids for the moving EM container, the permanent magnet, the permanent magnet embedment and the two pole pairs.

moving EM container

You can see the motion and force solution grids under this component. For example:

```
Xpos: pos displacement (m)
  1.00E-02   6.24E-03   -2.23E-03   -9.01E-03   -9.01E-03   -2.23E-03    6.24E-03
```

This is the displacement of the moving end of the container at discrete times of the cycle.

permanent magnet

You can see the linear flux densities and magnetic potential difference across the magnets as a function of time and position. For example:
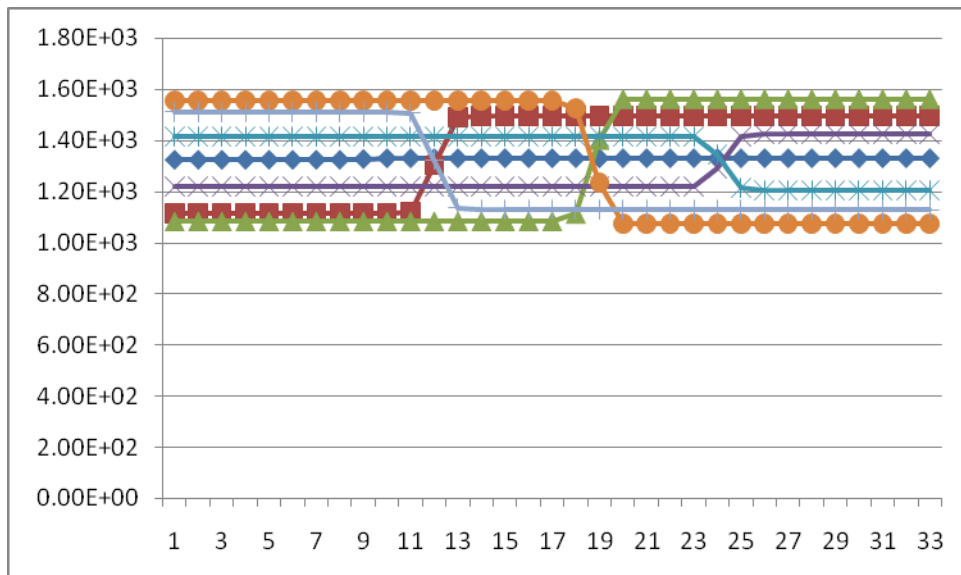
```
DeltaPsiPole: outer potential difference PsiPos-PsiNeg (A)
  1.32E+03   1.11E+03   1.08E+03   1.22E+03   1.42E+03   1.56E+03   1.51E+03
  1.32E+03   1.11E+03   1.08E+03   1.22E+03   1.42E+03   1.56E+03   1.51E+03
  1.32E+03   1.11E+03   1.08E+03   1.22E+03   1.42E+03   1.56E+03   1.51E+03
  1.32E+03   1.11E+03   1.08E+03   1.22E+03   1.42E+03   1.56E+03   1.51E+03
  1.32E+03   1.11E+03   1.08E+03   1.22E+03   1.42E+03   1.56E+03   1.51E+03
                                  ...
```

The columns correspond to discrete times of the cycle and the rows to computational nodes along the magnet length. The magnet is broken into NCell computational cells, with a node at the endpoints and middle of each cell. So every other row starting with the second row corresponds to a cell center.
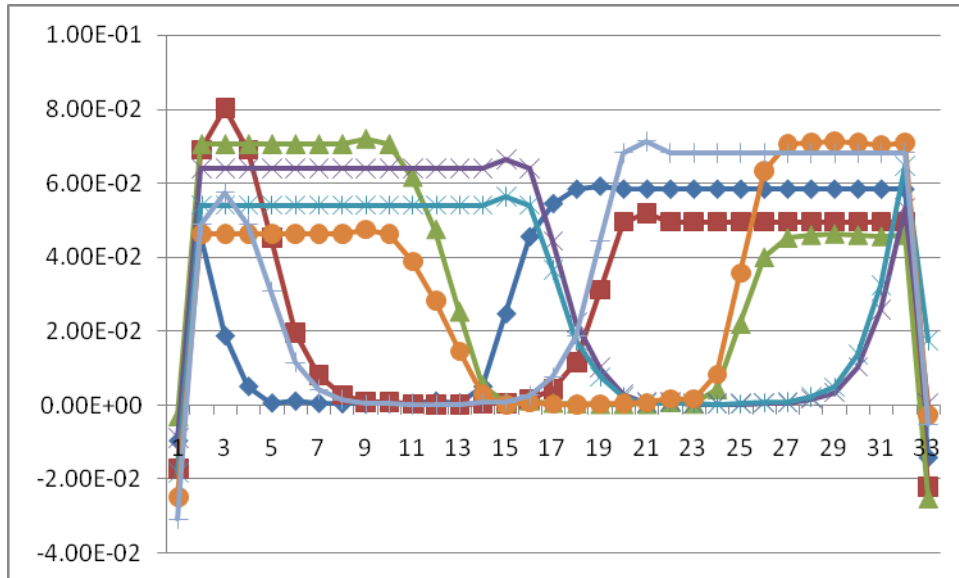
If you plot these values you see the outer potential difference looks like this:

Where each curve represents a different time and the horizontal axis corresponds to position along the magnet. The point where external potential shifts between pole 1 and pole 2 values shifts back and forth. That is because the data is in the magnet reference frame in which the outer poles are moving back and forth.

permanent magnet embedment

In the permanent magnet embedment you can see what is going on in the magnet itself. For example the linear magnetic flux density looks like this:

The magnetic flux drops off to near zero in the space separating the poles but not abruptly because Sage approximately models the fringing field outside the pole nominal boundaries.

When the magnet is within one pole or the other (two ends of plot) the magnetic flux is nearly constant except near the end where it drops off. This drop off is half way to the flux value for zero magnetication for the same outer potential difference and is necessary for the magnetic force calculation to be correct.

pole pairs 1 and 2

In the pole pairs the solution returns to time varying grids with lumped spatial quantities. You can see such things as the total magnetic flux, for example for pole 1 at discrete times of the cycle:

 Bflux: magnetic flux (Wb)
  -1.16E-03   -4.99E-04   5.37E-04   1.16E-03   9.18E-04   -3.13E-05   -9.46E-04