Sage Model Notes

PulseTubeSizer.ltc

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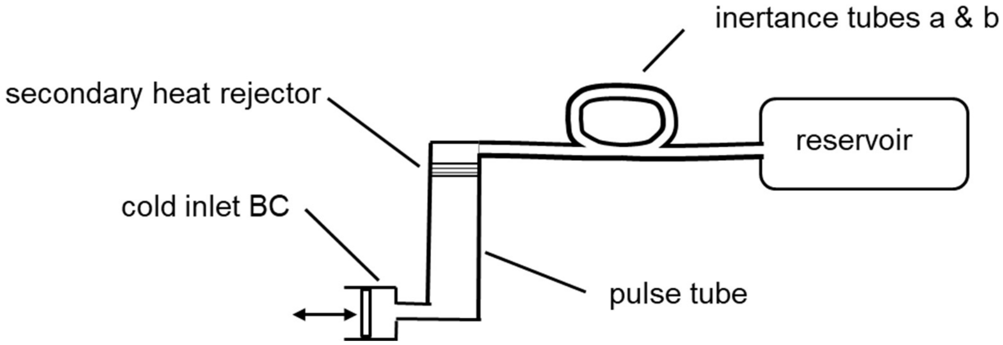
A stand-alone model of a pulse tube (buffer tube) with downstream pneumatic components designed to increase the phase lag of velocity (mass flow rate) relative to pressure, using fluid inertia, and boost the pressure amplitude relative to the velocity amplitude. In other words, to adjust the complex-valued acoustical impedance. The model schematic looks like this:



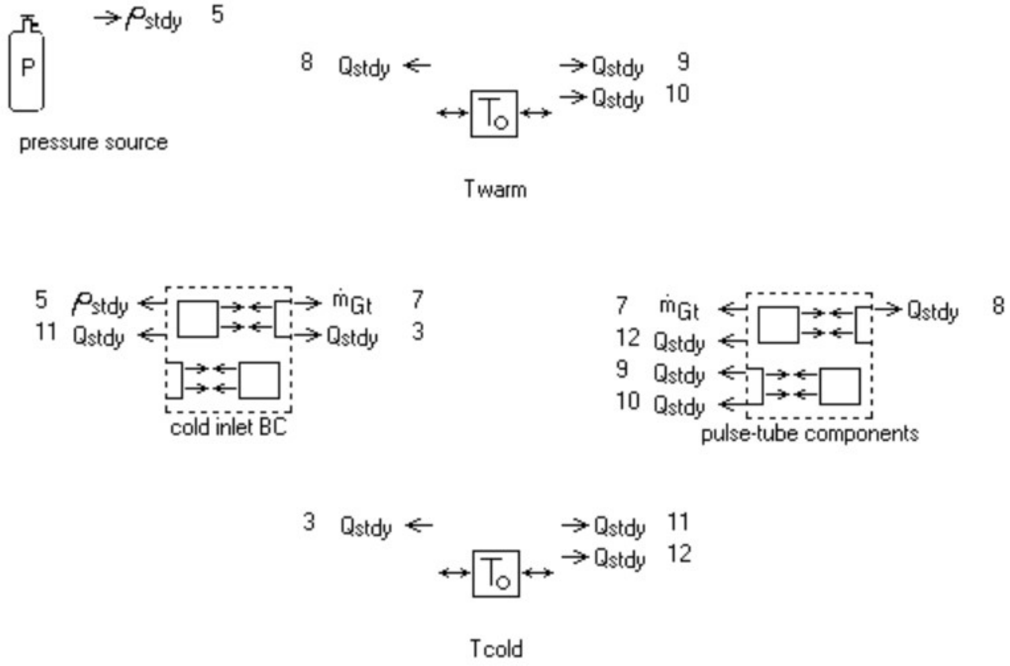
This model is a companion to **4KThreeStage_RegenSizer.ltc**. It is intended for designing the pulse tube and impedance-adjusting components separately from the full cryocooler model. The components can then be copied into a full cryocooler model to provide a feasible starting point for further optimization. See the model notes 4KThreeStage_RegenSizer.ptb for more discussion on this design strategy.

Note: Copy-and-pasting of model components in Sage does not use the Windows operating system clipboard. So you cannot copy from one instance of Sage to another instance. You have to copy from a source model, close that model and open the target model with the same Sage instance to paste the component(s).

The *cold inlet BC* component is an idealized piston / cylinder submodel anchored to the cold temperature. The mass flow rate amplitude and phase are defined as inputs. The pressure amplitude and phase are solved according to the state of the pneumatic circuit. The idea is to optimized the pneumatic circuit in order to satisfy pressure amplitude and phase constraints derived from a separate cryocooler model or simply to maximize the gross cooling power. More on that below.

Root Model

The root model defines the operating frequency and contains components representing the time-average pressure, cold and warm temperatures, along with two submodels:



Component *Twarm* anchors the warm end of the pulse tube along with the secondary heat rejector and reservoir components. Component *Tcold* anchors the cold end of the pulse tube as well as a heat transfer surface inside the *cold inlet BC* submodel.

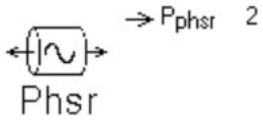
The root-level model contains the following summary outputs for the solved pressure amplitude and phase, the warm and cold heat rejections, PV power flow into the pulse tube and the gross cooling power, defined as the PV power flow minus the cold heat rejection:

```
Pamp             pressure amplitude cold end              1.438E+05
  PampExpander
Parg             pressure phase cold end                  7.998E+01
  PargExpander
QwarmRej         warm heat rejection                      1.810E+01
  QTwarm
QcoldRej         cold heat rejection                      1.408E+01
  QTcold
Wpv              PV power flow cold end                   7.215E+01
  -Wexpander
QgrossCooling    gross cooling power                      5.807E+01
  Wpv - QcoldRej
```

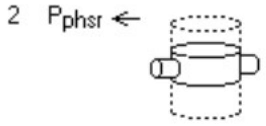
In a full cryocooler model the gross cooling power is superceded by the net cooling power, which is lower by the time-average helium enthalpy flow down the regenerator, the regenerator-wall thermal conduction loss and possibly other thermal losses.

Cold Inlet BC Submodel

This submodel contains a constrained piston to provide a prescribed volumetric displacement to a generic cylinder (variable-volume space) with a copper wall inside to anchor the temperature.



constrained piston



generic cylinder

There are some inputs associated with the cold inlet mass flow rate and some outputs that translate those inputs into swept volume amplitude for internal use:

Inputs

```
Inputs
  MdotAmp          mass flow rate amplitude (kg/s)          2.761E-02
  MdotPhase        mass flow rate phase (deg)               1.912E+01
  Rhom             time-average density (kg/m3)             1.340E+01
  Dexpander        effective piston diameter (m)            4.000E-02
  Angle90           (deg)                                   9.000E+01
```

Outputs

```
  VdotAmp          volumetric flow amplitude                2.061E-03
    MdotAmp / Rhom
  Vamp             volume amplitude                         1.093E-05
    VdotAmp / (2*Pi*Freq)
  Aexpander        effective expander area                  1.257E-03
    0.25*Pi * Sqr(Dexpander)
```

MdotAmp and MdotPhase correspond to the FRhoUA… outputs at the exit of the cold heat exchanger component of a full cryocooler model. Input Rhom corresponds to the time-mean gas density of that cold heat exchanger (most easily found by plotting the solution variables). It is important for calculating the equivalent volumentric flow amplitude Vamp. Input Dexpander is an artificial dimension that has nothing to do with the actual cryocooler but only the cold-inlet submodel implementation. Angle90 is just a convenient constant so that the components work right if you change the angle display units from degrees to radians.

The generic cylinder recasts inputs so its volume is always 50% larger than the swept volume amplitude and there is a lot of heat-transfer surface available to anchor the temperature.

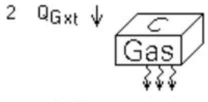
Recasts

```
  Volume = 1.5*Vamp
  Swet = 10*Aexpander
```

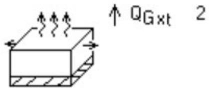
The constrained piston recasts amplitude, phase and frontal area (negative-facing area) according to the above inputs

```
  Xphase = MdotPhase - Angle90
  Xamp = VAmp / Aexpander
  A = Aexpander
```

The gas inside the generic cylinder exchanges heat with a copper thick-surface connected to the cold temperature.



cylinder-space gas

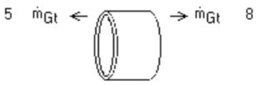


thick surface

That means the solved temperature will be close to the cold temperature, but for best model convergence the initial temperature should be set close to the cold temperature using the generic cylinder input

```
 Tinit           initial temperature (NonDim, K)         unit spline...
    (0.000E+00, 8.000E+01)
    (1.000E+00, 8.000E+01)
```

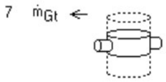
Pulse-Tube Components



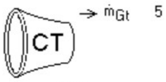
sec rej

ntrube a

ntrube b



reservoir

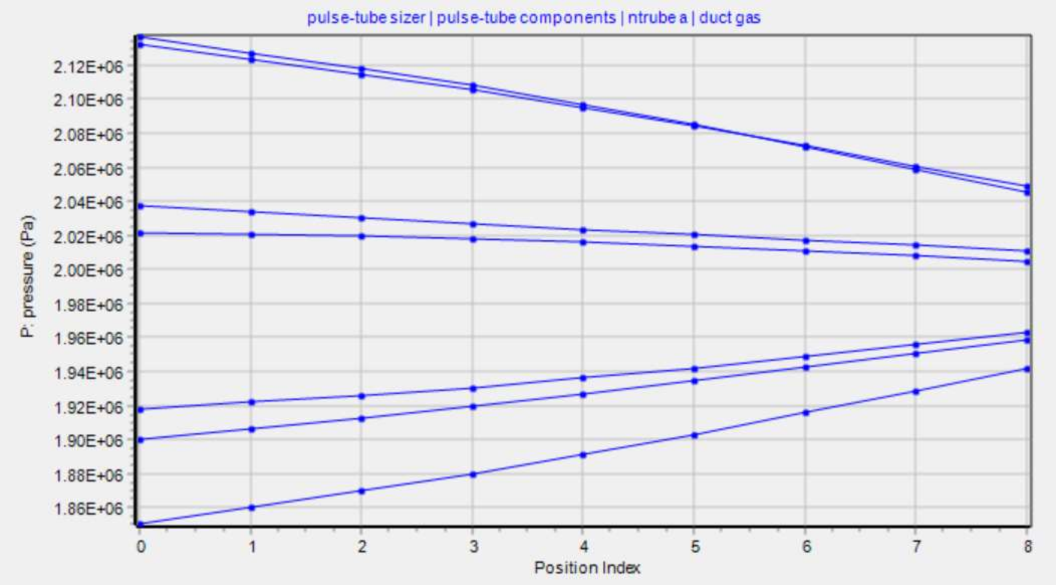


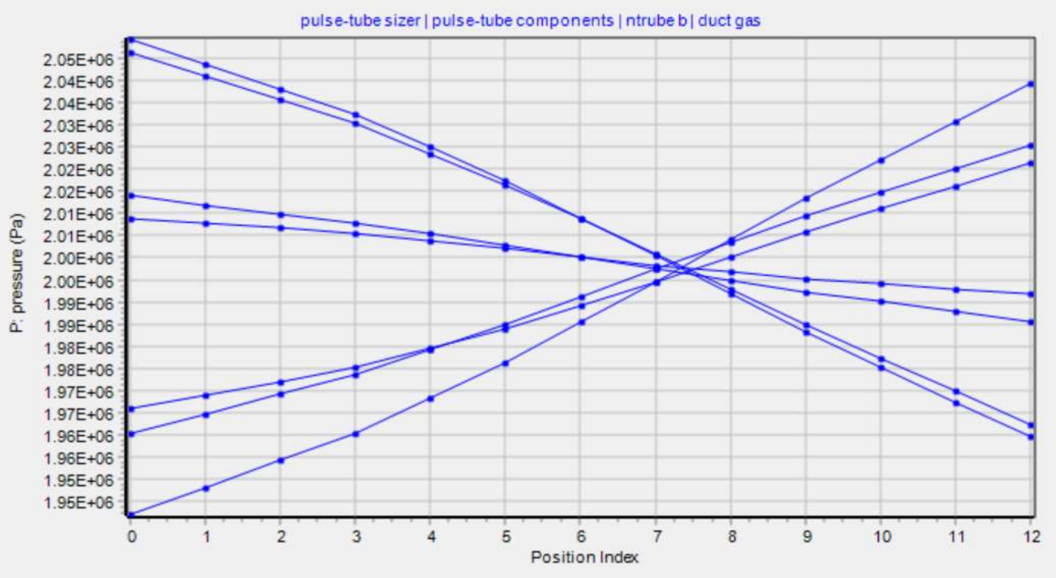
Ptube

The top row of components are anchored to the warm temperature. In the case of the *sec rej* and *reservoir* the anchoring is via heat-flow connectors to the *Twarm* temperature of the root model. In the case of the inertance-tube components *ntube a* and *ntube b* the anchoring is directly to an isothermal wall within because the lengths are too long to anchor by endpoint heat connections.

The wall of the Ptube component is anchored at both ends by heat-flow connectors to *Twarm* and *Tcold*.

The impedance-adjusting components are the two inertance tubes *ntube a* and *ntube b* connected to the reservoir (a volume large enough that its pressure fluctuation is relatively small). Compared to an orifice, the inertance tubes provide fluid inertia to retard the mass-flow-rate phase relative to pressure and also dissipate PV power via internal flow resistance. There are two of them as a stepwise approximation of a tapered quarter wave resonator with a pressure anti-node at the negative (left) end and a pressure node near the positive end. The diameter reduction at the negative end increases the pressure amplitude. The pressure solution in the two inertance tubes approximate an acoustic standing wave, as plotted below.





Note that the vertical and horizontal scales of the two plots are different. The pressure is actually continuous between the two. The pressure node occurs more-or-less upstream of the reservoir, depending on the reservoir volume. The larger the volume (lower its pressure amplitude), the closer the node to the reservoir.

The inertance tubes are rather long because the frequency is rather low and the speed of sound for helium relatively high.

ntube a

```
  Length            duct length (m)                          3.944E+00
  Dtube             tube internal diameter (m)               1.238E-02
```

ntube b

```
  Length            duct length (m)                          1.068E+01
  Dtube             tube internal diameter (m)               3.369E-02
```

Optimization

Optimized variables for this model are:

- Pulse tube length and diameters at two ends (assuming linear taper), subject to a maximum taper and minimum length constraint.
- Length and ID of the two inertance tubes, subject to the constraint that the pressure phase angle (root-level Parg) be no less than 80 degrees.

The optimization objective function is to maximize the gross cooling power QgrossCooling (PV power Wpv – cold thermal loss QcodRej). Another option would be to maximize the PV power flow (Wpv) but then the cold thermal loss might get out of hand. The pressure phase constraint is necessary to match the needs of the upstream regenerator in a full cryocooler model, namely that the pressure and mass flow rate should be in phase somewhere in the regenerator for optimal performance (giving most PV power flow for least thermal loss).

If the target pressure amplitude and phase are known, both Pamp and Parg could be constrained by equality constraints, but there is no guarantee that the optimizer will be able to satisfy those constraints within the limits of the pneumatic circuit. Fluid inertia (inertance) always comes with some amount of frictional dissipation (resistance) and volume (compliance). So while there may be some theoretical combination of inertance, resistance and compliance that will produce any given pressure amplitude and phase from a given mass flow rate boundary condition, it may not be possible in practice.