# *Sage Model Notes*

## 3KJTLoop.scfn

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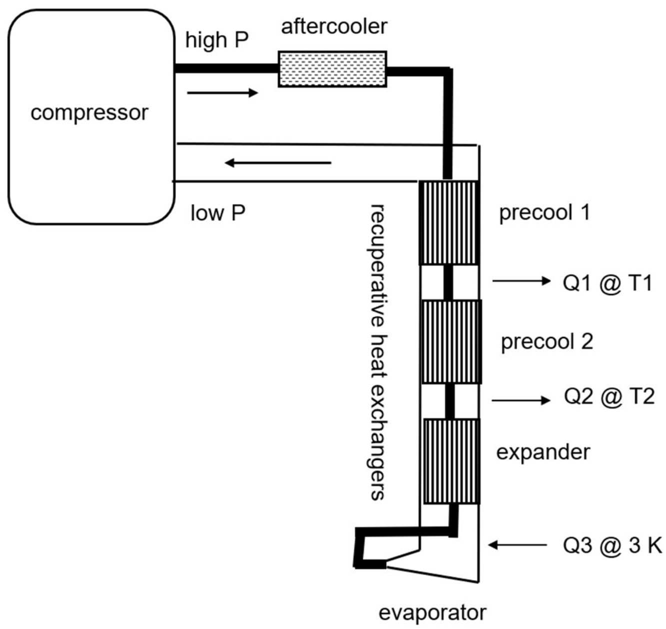
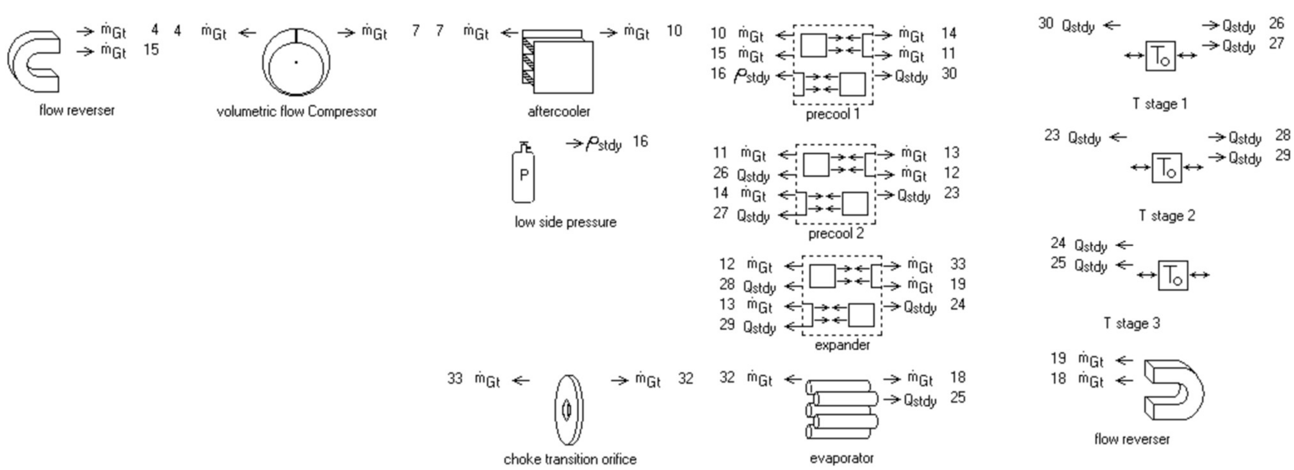
A model for a closed-cycle Joule-Thomson cryocooler operating between temperatures of 300 K and 3K, employing helium as the working gas.




The above sketch rendered as a Sage model looks like this:




This model borrows elements from two other sample models. From *HeatExchangers-CounterflowRecuperative.scfn* for the rucuperative heat exchangers used in the *precoo*l and *expander* submodels. And from *RefrigerationLoop* for the *volumetric flow compressor*, *choke transition* orifice and *evaporator* of the final stage.

The model is roughly organized in rows of progressively lower temperature:

**The top row** contains a *volumetric flow compressor* to drive the helium, an ambient temperature (300 K) *aftercooler* to remove the adiabatic heat of compression, a *precool 1* submodel within which is a recuperator anchored at the cold end to a heat exchanger, attached to the *T stage 1* heat sink, representing an attachment to an external cryocooler, at 60 K (current value). The *low side pressure source* of the second row logically belongs to this row. It provides the compressor suction-side pressure via attachment to the return-flow heat exchanger within the *precool 1* submodel. The flow reverser switches the return-flow direction so it can enter the suction side of the compressor.

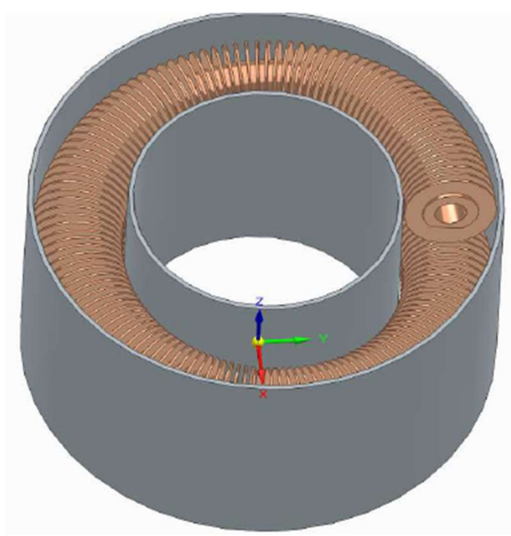
**The second row** contains a *precool 2* submodel within which is another recuperator in series with the one in the *precool 1* submodel, anchored at the cold end to a heat exchanger, attached to the *T stage 2* heat sink, representing an attachment to an external cryocooler, at 12 K.

**The third row** contains a *expander* submodel within which is another recuperator in series with the one in the *precool 2* submodel, anchored at the cold end to a heat exchanger attached to the *Tstage 3* heat sink, representing an attachment to a heat load at 3 K.

**The fourth row** contains a choke transition orifice, which implements a throttling process to reduce the pressure at the exit of the high-pressure tube of the *expander* recuperator (the *expander tube*) from 0.8 bar to 0.2 bar, the saturation pressure of helium at 2.9 K. It also contains an *evaporator* heat exchanger, attached to the *T stage 3* heat source, whose purpose is to vaporize the helium in order to produce latent-heat cooling. In this model only some of the vaporization occurs in the *evaporator* (quality increasing from 0.60 to 0.64) with the remaining vaporization (to quality 1.0) occurring in the return stream of the *expander* recuperator. The flow reverser diverts the helium flow back up that return stream.

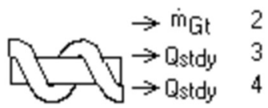
# The Recuperators

The two *precool* and the *expander* submodels contain the same style recuperative heat exchangers, copied from the *HeatExchangers-CounterflowRecuperative.scfn* sample model. They model externally finned tubes (the high-pressure tubes, 1 tube per recuperator) wrapped helically so that the spaces between the fins form the return path.



In this model the tube and fins are stainless steel instead of copper. The **shroud materials are not part of this model,** presuming they would be made from insulating material and not change the performance much. Thermal conduction through the tube fins from one end of the helical envelope to the other is modeled as if there is a continuous metallic path for the full envelope length. Reducing thermal conduction is the main reason the fins are made of stainless steel rather than copper.  Thermal conduction through the spiral tube is based on the unwrapped total tube length.

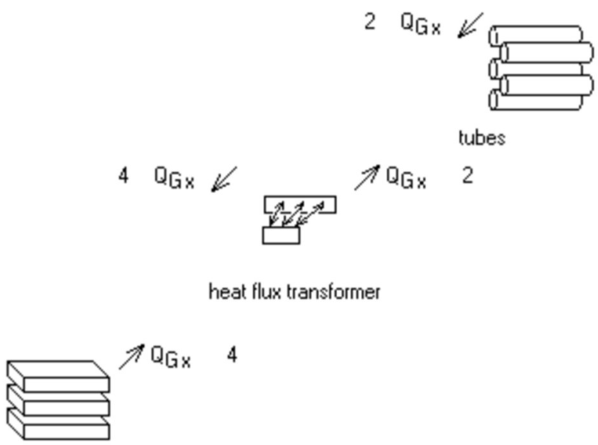
Here is how the above is rendered in the Sage *precool 1* submodel :




circumferential finned tube recuperator


cold HX

The components inside the *circumferential finned tube recuperator* are as follows:




The documentation for the *HeatExchangers-CounterflowRecuperative.scfn* sample model explains the heat transfer process between the *tubes* and *circumferential fins*.

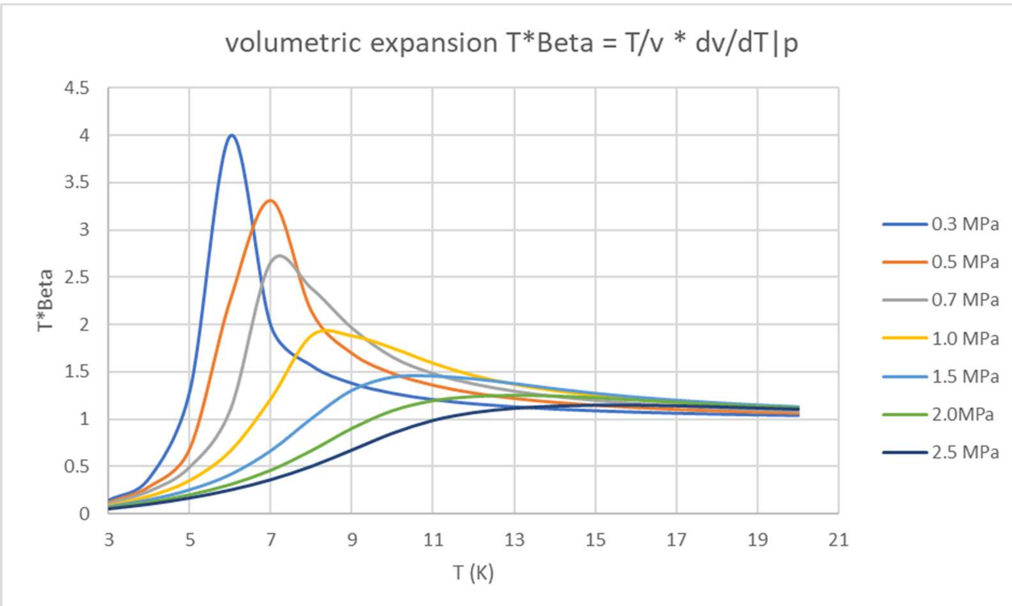
# The Orifice

The *choke transition orifice* may or may not exist physically in hardware but it can help the Sage solution to converge. It is the only component in the model that can allow the model to avoid the problems associated with approaching the speed of sound. As helium flows in a tube from high to low pressure the velocity increases in inverse proportion to the density. If velocity approaches the sonic velocity at the exit the pressure cannot decrease further to the low side pressure — the flow is choked — and convergence becomes more problematic. An orifice allows the pressure to change discontinuously from high to very low pressure without the convergence problems. It imposes a pressure drop into the solution, corresponding to the shock-boundary expansion of the sonic flow leaving the orifice at the *expande*r tube exit pressure. The Sage User's Guide explains this in more detail.

# Joule-Thomson Principles

Operating a JT cooling cycle between 300 K and 3 K requires help because JT cooling is based on the principle that enthalpy depends on pressure and a reduction of pressure at constant enthalpy lowers the temperature. This requires non-ideal gas behavior and above around 20 K helium approximates an ideal gas. In thermodynamic terms the quantity $T\,\beta$ must be greater than one for JT cooling

The following plot shows $T\beta$ for helium 4, as a function of temperature, at various pressures.

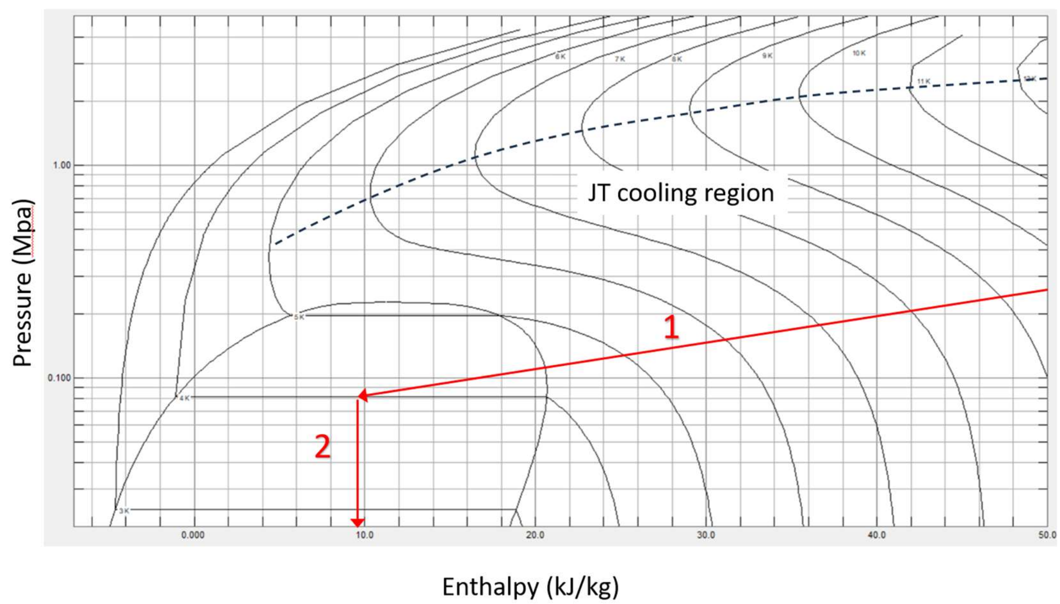



T*Beta plot from NIST Refprop software

This plot means that the lion's share of JT cooling is confined to the coldest *expander* stage and the purpose of the recuperative heat exchangers in the upper two *precool* stages is only to lower the helium temperature from 300 to around 13 K as efficiently as possible. This is accomplished by the recuperators where heat transfer between the high-pressure tubes and low-pressure return flow cools the high-pressure gas while reheating the low-pressure gas. Low-temperature heat sinks at the cold end of each high-pressure tube compensate for recuperator ineffectiveness.

In another view from the kaleidoscope, the next plot illustrates JT principles from the point of view of pressure and enthalpy. Processes at constant enthalpy are vertical lines. Depending on the region of the plot, lowering pressure at constant enthalpy (throttling) will either increase or decrease temperature with the dividing curve between the two passing through the points where the isotherms have vertical slopes (dotted curve). Drawn on the plot are two paths to 3 K cooling that this model takes: (1) within the high-pressure tube of the *expander* recuperator, reducing temperature from 12.5 K down to around 4 K, entering the two-phase region just above 4 K, and (2) under a throttling process in the orifice, dropping the temperature just below 3 K. Process 1 combines JT cooling with the enthalpy decreases with temperature produced by the helium in the high-pressure tube rejecting heat to the low pressure return stream.

Pressure vs Enthalpy plot helium




P-h plot from NIST Refprop software

# User Defined Variables

Documented below are some key user-defined inputs and outputs. As usual the best way to explore the detailed relationships between user-defined inputs, outputs and recast inputs is through the Explore Custom Variable menu item in the Sage interface.

### Root Model

```
Inputs
  Tstage1          stage 1 temperature (K)                  6.000E+01
  Tstage2          stage 3 temperature (K)                  1.208E+01
  Tstage3          stage 3 temperature (K)                  3.000E+00
Outputs
  Win              mechanical input power                   1.999E+02
    -Wcomp
  Wstage1          precooler power input estimate           9.981E+01
    Qrej1*Power(300/Tstage1, 1.9)
  Wstage2          precooler power input estimate           5.884E+02
    Qrej2*Power(300/Tstage2, 1.9)
  Qrej1            1st stage heat rejection                 4.690E+00
    -Qnet1
  Qrej2            2nd stage heat rejection                 1.315E+00
    -Qnet2
  Qlift3           3rd stage heat lift                      5.000E-02
    Qnet3
```

These specify the temperatures of the three temperature sources and the heat flows to/from them. The model is configured so that all the heat flows of interest come from connections to the three heat sources. The exception is the heat rejected to the aftercooler, which goes to an *independent line heat source* within that component.

`Win` is the mechanical power required by the *volumetric flow compressor*, according to the adiabatic compressor equation. `Wstage1` and `Wstage2` are estimates for the mechanical power required for an external cryocooler to remove `Qrej1` and `Qrej2`. These estimates are based on Sage simulations of multi-stage stirling-cycle cryocoolers. See *Estimating Precooler Power* below.

**Submodels**

All submodels contain the following inputs, used to recast inputs of the individual heat exchangers within the submodel. These recasts follow the recommendations of the *HeatExchangers-CounterflowRecuperative.scfn* sample model. The particular values below are from the *precool 1* submodel.

```
Inputs
  IDtube           tube ID (m)                                 2.000E-03
  TwallTube        tube wall thickness (m)                     2.500E-04
  ODfin            fin outer diameter (m)                      5.000E-03
  ThkFins          fin thickness (m)                           2.500E-04
  SpacingFins      fin to fin spacing (m)                      1.000E-03
  DmeanShroud      annular shroud mean diameter (m)            2.000E-02
  NturnsShroud     number turns tube in shroud (NonDim)        3.000E+01
Outputs
  ODtube           tube outer diameter                         2.500E-03
    IDtube + 2*TwallTube
```

***Expander tube* and *Evaporator***

The gas components inside the *expander* and *evaporator* have some useful outputs pertaining to the two-phase state at the inlet and outlet. The following are from the *evaporator*:

```
  QualNeg          vapor quality neg bnd                       5.965E-01
    Gas.Qual(RhomNeg, TNeg)
  QualPos          vapor quality pos bnd                       6.378E-01
    Gas.Qual(RhomPos, TPos)
```

On the *quality* scale 0 means pure liquid and 1 pure vapor.

# Compressor

The *volumetric flow compressor* implements a steady-flow adiabatic pressurization process according to the following inputs:

```
  Aflow            mean flow area (m2)                         1.000E-04
  Tinit            initial temperature (K)                     3.000E+02
  Efficiency       adiabatic efficiency (NonDim)               8.000E-01
  Rclearance       relative clearance volume                   1.000E-02
  Vdot             volumetric flow rate (m3/s)                 1.553E-03
```

`Vdot` is the equivalent of the per-stroke volumetric displacement of a piston-type compressor times the compressor operating frequency.

`Rclearance` represents the ratio of the minimum dead volume at the end of the compression stroke compared to the volume at the beginning.

In this model the suction pressure is 0.2 bar and discharge pressure 3.9 bar. **The pressure ratio is 20** (discharge/suction), which is rather high. To achieve this pressure ratio requires a compressor volume ratio given by the adiabatic equation

$$\frac{V_{min}}{V_{max}} = \left(\frac{P_{min}}{P_{max}}\right)^{1/\gamma}$$

Where $\gamma$ = 1.67 for helium. Doing the math, $V_{min}/V_{max} = 0.17$ for this pressure ratio. In other words, Rclearance can be at most 0.17. Higher than that will result in no flow.

**The adiabatic temperature increase** follows the relationship

$$\frac{T_{max}}{T_{min}} = \left(\frac{P_{max}}{P_{min}}\right)^{\frac{\gamma-1}{\gamma}}$$

Which works out to $T_{max}/T_{min} = 3.3$ for this pressure ratio. With $T_{min} = 290$ K at the inlet this means, $T_{max} = 960$ K at the outlet (690 C), which is red hot. The aftercooler heat exchanger reduces the helium temperature back to a more reasonable temperature of 314 K. Evidently the practical design of the compressor is important, but that is beyond the scope of this model.

# Optimization

In this model the compressor volumetric flow rate is optimized to produce 50 mW cooling at 3 K, with minimum compressor mechanical power input. The two precooler temperature sources are also optimized to minimize the estimated mechanical power inputs for the cryocooler anchoring those temperatures, except that the stage 1 temperature is constrained to be at least 60 K. The optimized results are in the above *Root Model* summary.

A summary of the optimization specification according to the Tools | Explore Optimization dialog is:

```
Objective Function
  Minimize Win  + Wstage1 + Wstage2


OPTIMIZED VARIABLES                    SUBJECT TO CONSTRAINTS

JT cooler
  Tstage1                              Qlift3 = 0.05
  Tstage2                              Tstage1 > = 60

4 volumetric flow compressor
  Vdot
```

# Convergence after Reinitialization

There is no need to reinitialize the model after you change inputs. In fact, it is a bad idea because convergence is much more likely when starting from a previous converged solution.

In the event you need to reinitialize the model for some reason you will most likely have to gradually coax the model back to a converged state. Here are some steps that have been known to work for this model:

1. Reduce compressor input `Vdot` to a small value like 1.0E-07
2. Make sure root input `Pnorm` is the same value as the charge pressure in the pressure source

3. Set root inputs `Tstage2` and `Tstage3` to 20 K, which are their initial values according to Tinit inputs
4. Reinitialize the model and Solve. If it doesn't converge adjust *Vdot*, *Pcharge* and *Pnorm* inputs and try again.
5. Once converged, gradually reduce `Tstage2` and `Tstage3` to final values
6. Then gradually increase Vdot in order to increase the cooling power

# Estimating Precooler Power

*The following notes are taken from a D. Gedeon memo of 2021*

In an ideal pulse-tube cycle the compressor work $W$ at ambient temperature $T_w$ to produce a cooling power $Q_c$ at temperature $T_c$ is simply

$$W = \frac{T_w}{T_c} Q_c$$

This follows from the isentropic condition $Q_w/Q_c = T_w/T_c$ and equating the required power input $W$ with the warm heat rejection $Q_w$ according to the first law of thermodynamics. So the incremental cost in compressor PV power scales in proportion to the temperature ratio over which the heat is lifted.

What about in a non-ideal pulse-tube cooler? In this case it is sensible to postulate that the ideal isentropic heat-lift condition is replace with the non-ideal condition $Q_w/Q_c = (T_w/T_c)^\alpha$, where $\alpha$ is some exponent greater than one. The reason this is sensible is that it applies to individual stages of a multi-stage cooler as well as the entire cooler. For example, in a two-stage cooler the second stage considered alone would lift heat $Q_c$ at $T_c$, and reject heat $Q_1$ to the intermediate stage at $T_1$, according to

$$Q_1 = Q_c(T_1/T_c)^\alpha$$

The first stage considered alone would lift this same heat $Q_1$ at $T_1$ and reject $Q_w$ at ambient temperature $T_w$, according to

$$Q_w = Q_1(T_w/T_1)^\alpha$$

Substituting for $Q_1$ in the first equation using the second equation shows that the overall efficiency has the same form.

$$Q_w = Q_c(T_1/T_c)^\alpha(T_w/T_1)^\alpha = Q_c(T_w/T_c)^\alpha$$

Sage simulations indicate that $\alpha = 1.9$ is resonably accurate for the efficiency exponent in a realistic PTR. So the compresor power increment due to heat load $Q_j$ on stage $j$ is

$$\Delta W_j \approx Q_j\left(T_0/T_j\right)^{1.9}$$