Sage Model Notes

SplitCycleCoolerTwoStage.scfn

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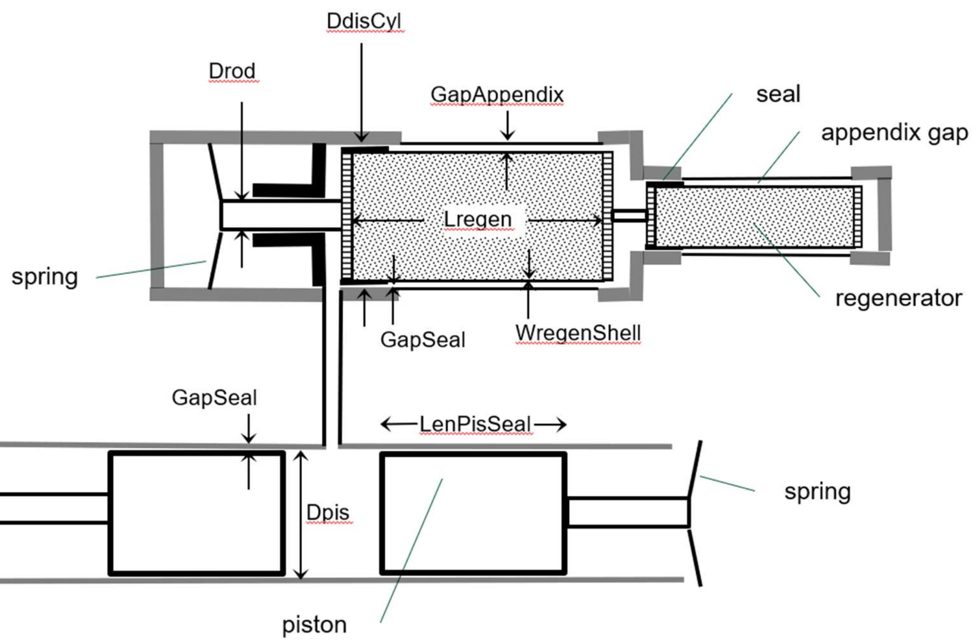
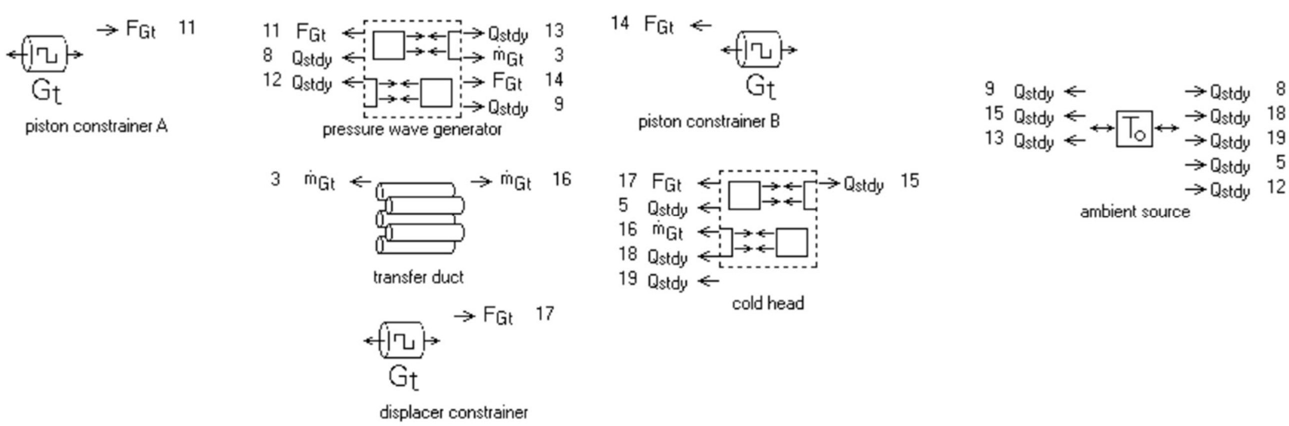
A model for a split-cycle stirling two-stage cryocooler providing cooling at 60 K and 12 K, arranged something like this:



In the Sage model the *pressure wave generator* and *cold head* are in separate submodels with a *transfer duct* connecting the two:



A number of user-defined variables correspond to the dimensions of the above drawing. The root model defines the design-point cold stage temperatures, piston and displacer motions and also some summary *out*puts:

```
Inputs
  Tstage1          stage 1 temperature (K)                   6.000E+01
  Tstage2          stage 2 temperature (K)                   1.200E+01
  XpAmpDesign      design piston amplitude (m)               5.000E-03
  XpPhsDesign      design Xp phase (rad)                     1.571E+00
  XdAmpDesign      design displacer amplitude (m)            3.043E-03
  XdPhsDesign      design Xd phase (rad)                     2.549E+00
Outputs
  Wp               net PV power                             -2.500E+02
    pvPWG + pvDisCmp+pvExp1 + pvExp2 +pvDisBuff
  Qlift1           stage 1 heat lift                         8.243E+00
    QT1
  Qlift2           stage 2 heat lift                         3.873E-01
    QT2
  Qrej             total heat rejection                     -2.586E+02
    QpsA + QpsB +Qgd+Qrods+Qcmpd+QTrej
```

The *pressure wave generator* submodel contains these inputs:

```
Inputs
  Dpis             piston diameter (m)                       4.210E-02
  LenPisSeal       piston clearance seal length (m)          2.500E-02
  SwetBuff         buffer space wetted surface (m2)          3.000E-02
  VolBuff          buffer space volume (m3)                  3.000E-04
  Nsprings         number of springs each side (NonDim)      2.000E+00
  Kstatic          static spring stifffness (N/m)            9.613E+03
  Fnat             1st mode resonant frequency (Hz)          2.000E+02
  Mpis             piston/alternator recip mass (kg)         7.500E-01
  GapSeal          clearance seal gap (m)                    1.250E-05
  EccenSeal        eccentricity seal 0 to 1 (NonDim)         5.000E-01
```

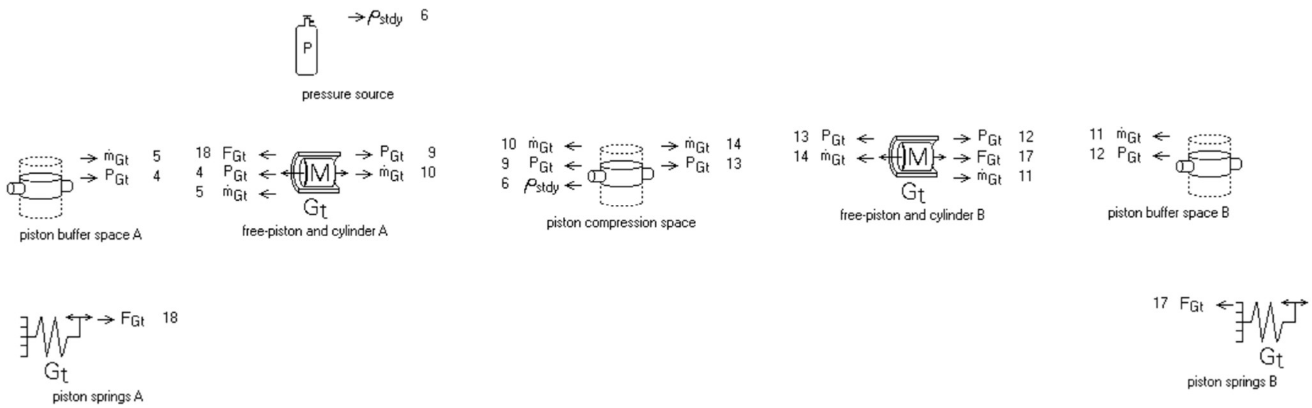
The cold head submodel contains these inputs:

```
Inputs
  Drod             displacer rod diameter (m)                7.345E-03
  DdisCyl1         displacer cylinder ID (m)                 3.096E-02
  DdisCyl2         displacer cylinder ID (m)                 1.311E-02
  Lregen1          regenerator length (m)                    5.393E-02
  Lregen2          regenerator length (m)                    1.772E-02
  GapAppendix1     radial appendix gap (m)                   1.000E-04
  GapAppendix2     radial appendix gap (m)                   1.000E-04
  GapSeal          clearance seal gap (m)                    1.250E-05
  EccenSeal        eccentricity seal 0 to 1 (NonDim)         5.000E-01
  AllowStressPwall pressure wall allowable stress (Pa)       1.300E+08
  WallMin          min pressure wall thickness (m)           1.500E-04
  WregenShell      regenerator shell thickness (m)           5.000E-04
  Mdis             displacer recip mass (kg)                 1.500E-01
```

Most of these variables are used to recast built-in inputs of the components inside the submodels as explained below. As always, using the Tools | Explore Custom Variables dialog is the best way to trace these relationships and understand how the model works.

Pressure Wave Generator Submodel

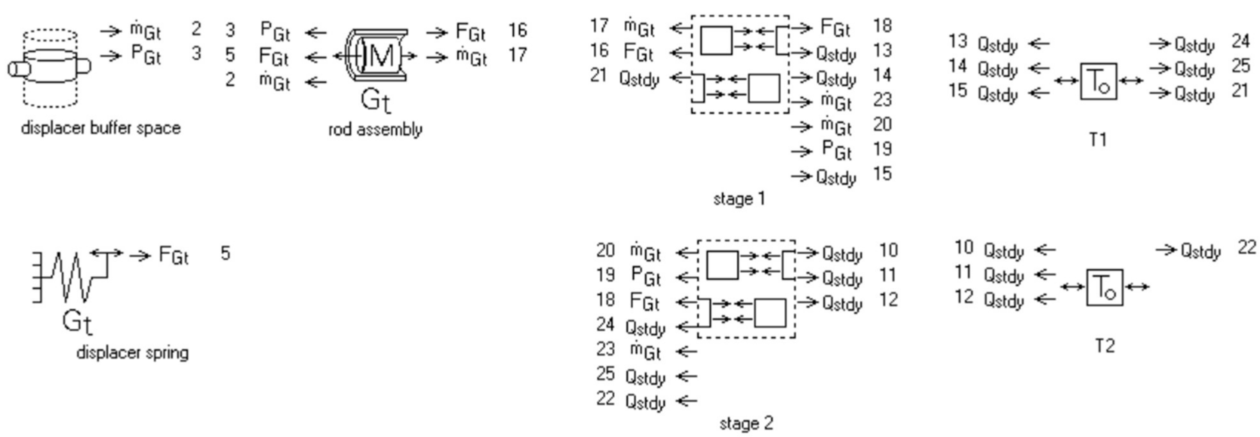
This is essentially the *PressureWaveGenerator.scfn* sample model without the actuator part:



It simulates an opposed-piston compressor driven by *piston constrainer* components in the root model. See the *PressureWaveGenerator.scfn* documentation for more details.

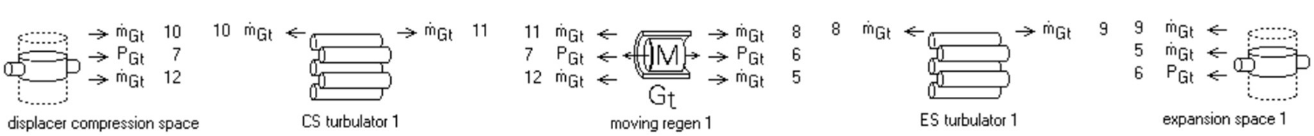
Cold Head Submodel

Squint carefully and you can see the cold-head layout of the above drawing in this Sage rendering:

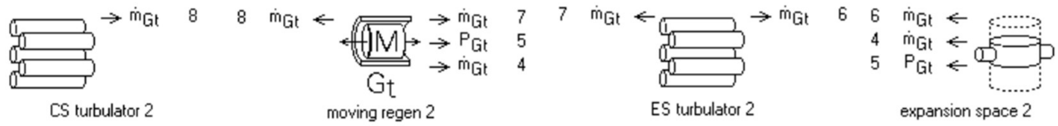


The *stage 1* and *stage 2* submodels contain similar components with inputs recast in terms of user-defined variables with “1” and “2” suffixes.

stage 1



stage 2



Heat rejection takes place in the *displacer compression space* and cooling in *expansion space 1* and *expansion space 2* as the result of turbulent-enhanced heat transfer resulting from incoming flow from the *turbulator* components.

Turbulators

The turbulators represent the perforated walls at either end of the moving regenerator canisters. They serve to hold the regenerators in place and also increase the velocity (kinetic energy) of outflowing helium to enhance the heat transfer in adjoining spaces. These ideas along with some notes on hole patterns are explained in more detail in the documentation for sample model *HeatExchanger-JetImpingement.scfn*.

In the table below the hole numbers are just rough estimates, the hole diameters are optimized and the open area fraction is a side calculation of interest.

	Number holes	Diameter holes (mm)	Open area fraction
CS turbulator 1	36	1.7	0.12
ES turbulator 1	48	1.1	0.06
CS turbulator 2	12	2.2	0.35
ES turbulator 2	12	2.2	0.35

The number of holes should be adjusted according to the hole pattern of an actual design after which the hole diameters can be reoptimized.

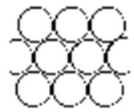
Cold turbulator note *ES turbulator 2* can produce solution convergence problems because of difficulty solving gas density (and therefore temperature) at a time nodes where the mass flow rate is near zero and there is a significant adiabatic heating or cooling due to a high pressure rate of change. This occurs because heat transfer to the hole solid boundary surface is too low and the solid specific heat is also very low. To alleviate this potential problem the input `KmultBnd` is set to 1.0 in the ES turbuilator 2 gas and also in the gas domains of the adjoining *moving regen 2* and *expansion space 2* components. This allows thermal conduction between the adjoining gas domains to stabilize the turbulator temperature.

Regenerators

The stage 1 regenerator is a relatively high porosity stainless steel random-fiber matrix, with optimized porosity. Unlike woven-mesh regenerators which have porosities in the range of 0.65 – 0.70, depending on the weave pattern, random fiber structures are readily fabricated with porosities up to 0.90 or higher. For this model the optimal porosity is 0.81


random fiber matrix

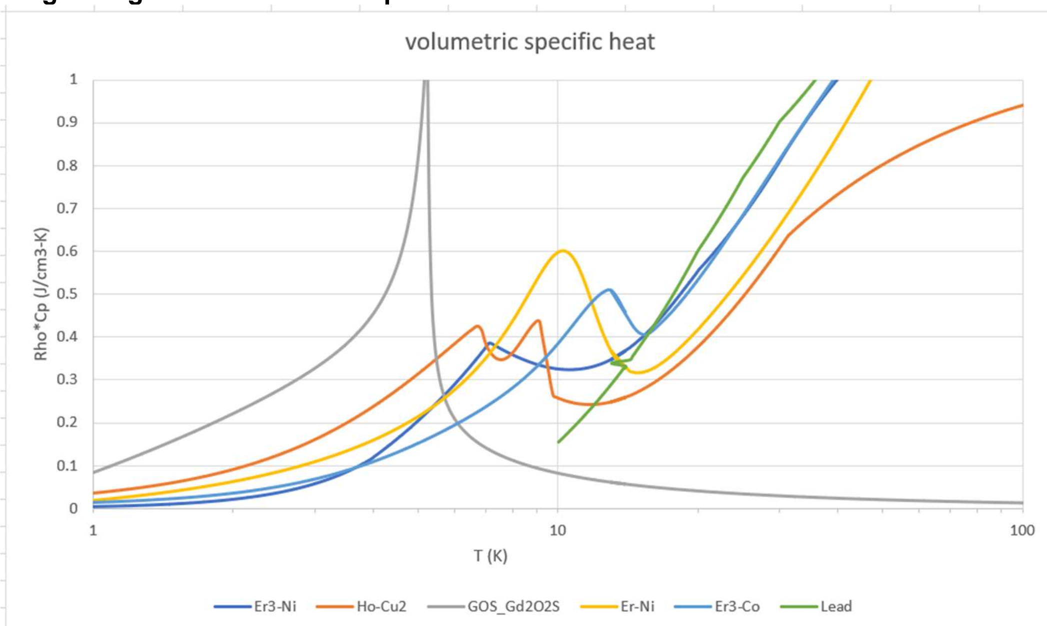
The stage 2 regenerator is a packed bed of lead particles:



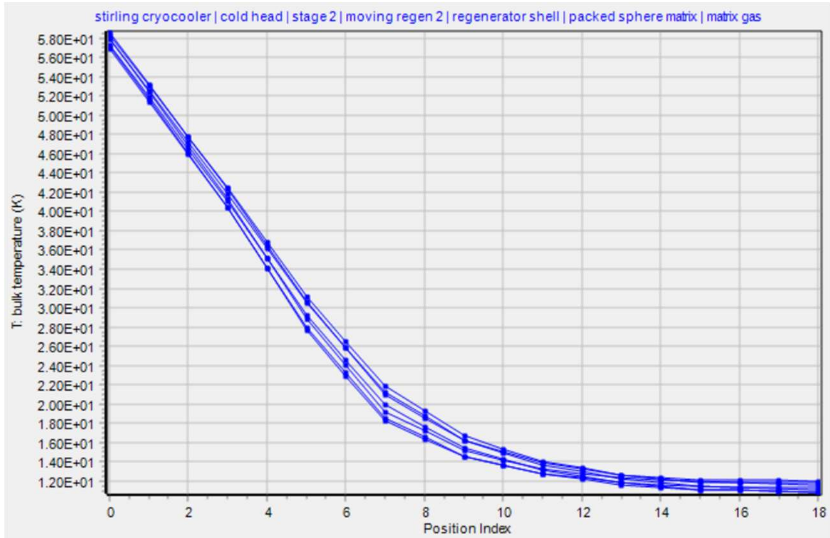
packed sphere matrix

This choice maximizes the ratio of solid-to-helium heat capacity at the low temperature end of the regenerator (12 K). The plot below compares specific heats for lead along with some other low-temperature regenerator materials. 12 K is about the lower limit for lead as a contender.

Stage 2 regenerator material options



The helium density is another consideration. The lower the density (pressure) the greater the ratio of solid-to-helium heat capacity. The current model uses 1 MPa helium pressure which seems to work reasonably well according to this plot of helium temperature vs position:



The diminishing slope toward the cold end is typical of cold regenerators. The spacing of the curves shows the variation of helium temperature at different times of the cycle.

Optimization

This model is optimized to maximize *stage 2* heat lift (`Qlift2`) with mechanical power input (`Wp`) constrained to 250 W. The *stage 1* heat lift (`Qlift1`) is constrained according to an empirical relationship designed to equally apportion the mechanical power input between `Qlift1` and `Qlift2`:

$$Q_1/Q_2 = (T_1/T_2)^{1.9}$$

This relationship is based on Sage simulation results from pulse-tube cryocoolers over a range of temperatures and is explained in more detail in the documentation for sample model *3KJTLoop.scfn*.

Here is the complete optimization summary produced by the Tools | Explore Optimization menu item:

```
Objective Function
  Maximize Qlift2

OPTIMIZED VARIABLES                 SUBJECT TO CONSTRAINTS

stirling cryocooler
  XdAmpDesign                       Wp = -250
  XdPhsDesign                       Qlift1  = Qlift2 *
Power(Tstage1/Tstage2, 1.9)
                                    Fspring = 0
                                    F.Cos.1 = 0
                                    F.Sin.1 = 0
                                    Qlift2 > = 0.1


1 pressure wave generator
  Dpis
  Kstatic

3 cold head
  Drod
  DdisCyl1
```

```
  DdisCyl2
  Lregen1
  Lregen2

3.2  displacer spring
  Kstatic

3.4 stage 1

3.4.2 CS turbulator 1
  Dtube                                Dtube > = 1.0e-3

3.4.3 moving regen 1

3.4.3.2 regenerator shell

3.4.3.2.1 random fiber matrix
  Porosity

3.4.4 ES turbulator 1
  Dtube                                Dtube > = 1.0E-3

3.5 stage 2

3.5.1 CS turbulator 2
  Dtube                                Dtube > = 1.0E-3

3.5.3 ES turbulator 2
  Dtube                                Dtube > = 1.0E-3
```

Free Running Dynamics

The model currently runs with piston and displacer motions constrained by the three piston constrainer components at the root model level — two for the opposed pistons of the *pressure wave generator* and one for the displacer of the *cold head*.

Part of the above optimization specification implements a tuning process that adjusts

1. The piston spring so that the actuator it might eventually be connected to has only to deal with work-related forces (in phase with piston velocity), and
2. The displacer rod and spring so the displacer will run at the same amplitude and phase when the displacer constrainer is disconnected.

The parts of the optimization that do this are:

```
OPTIMIZED VARIABLES                    SUBJECT TO CONSTRAINTS

1 pressure wave generator
 Kstatic                               Fspring = 0

3 cold head
  Drod                                 F.Cos.1 = 0
  Kstatic                              F.Sin.1 = 0
```

In the case of the *pressure wave generator* the variable *Fspring* comes from piston constrainer A, which defines these outputs:

```
  Fwork            work component of force                  1.592E+02
    -(F.Cos.1 * FX.Sin.1 - F.Sin.1*FX.Cos.1) / FX.Amp.1
  Fspring          spring component of force                5.342E-05
    -(F.Cos.1 * FX.Cos.1 + F.Sin.1*FX.Sin.1) / FX.Amp.1
```

The spring component of force is the component of the boundary force in phase with the position phasor, according to the standard geometrical interpretation of the vector dot product. The work component is the component of the boundary force perpendicular to the position phasor (i.e. in phase with the velocity).

In the case of the *cold head* displacer `F.Cos.1` and `F.Sin.1` come from the *displacer constrainer*. They are built-in outputs for the phasor components of the boundary force. Constraining both to zero means that the *displacer constrainer* imposes zero first-harmonic force on the displacer reciprocating mass.