

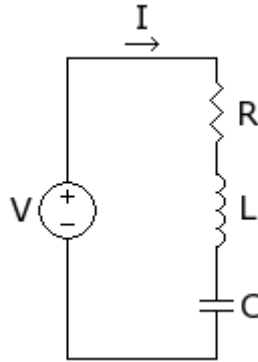
# Sage Model Notes

## SeriesRLCircuit.stl

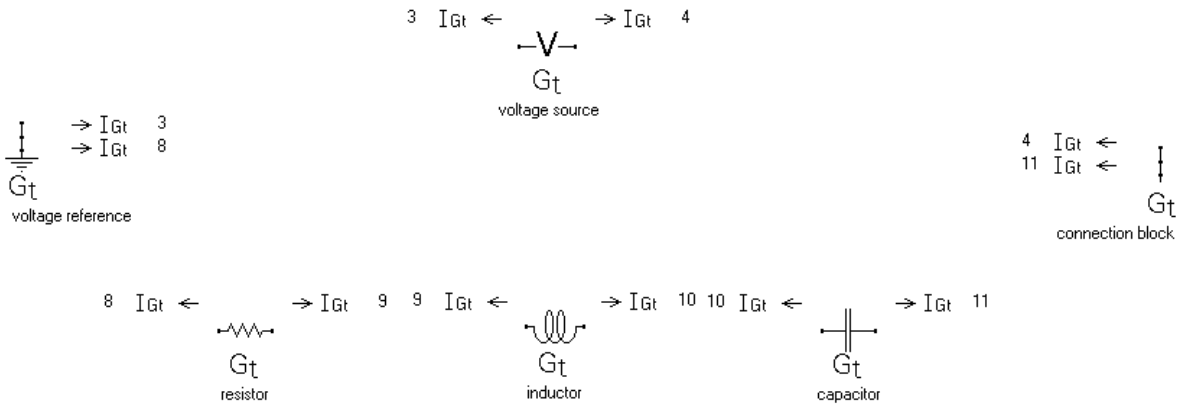
D. Gedeon  
16 December 2011

A model for a series RLC (resistor-inductor-capacitor) electrical circuit using Sage simple electrical components introduced in version 9.

In conventional electrical circuit notation the circuit schematic for this model is represented by this picture borrowed from Wikipedia:



The equivalent Sage model looks like this:



The model is set up to demonstrate how the amplitude and phase of the current through the RLC leg of the circuit responds to frequency. What drives the model is the constant voltage amplitude and phase of the *voltage source*, set by this inputs:

### Inputs

FDeltaV      Vpos-Vneg (V, deg)      0.000E+00...  
( 1.000)E+01 Amp  
( 0.000)E+00 Arg

The frequency of the driving voltage is set by the root level input *Freq*.

For convenience the impedance values of the three electrical components (resistor, inductor and capacitor) are recast in terms of three user-defined inputs at the root level:

**Inputs**

Rcirc circuit resistance (ohm) 1.000E+01  
 Lcirc circuit inductance (H) 5.000E-02  
 Ccirc circuit capacitance (F) 5.000E-05

One reason for doing this is so that the theoretical resonant frequency (below) can be calculated at the root model level as a user-defined output.

**FreqResonance** resonant frequency 1.007E+02  
 $\text{Sqrt}(1/(\text{Lcirc} * \text{Ccirc})) / (2 * \text{Pi})$

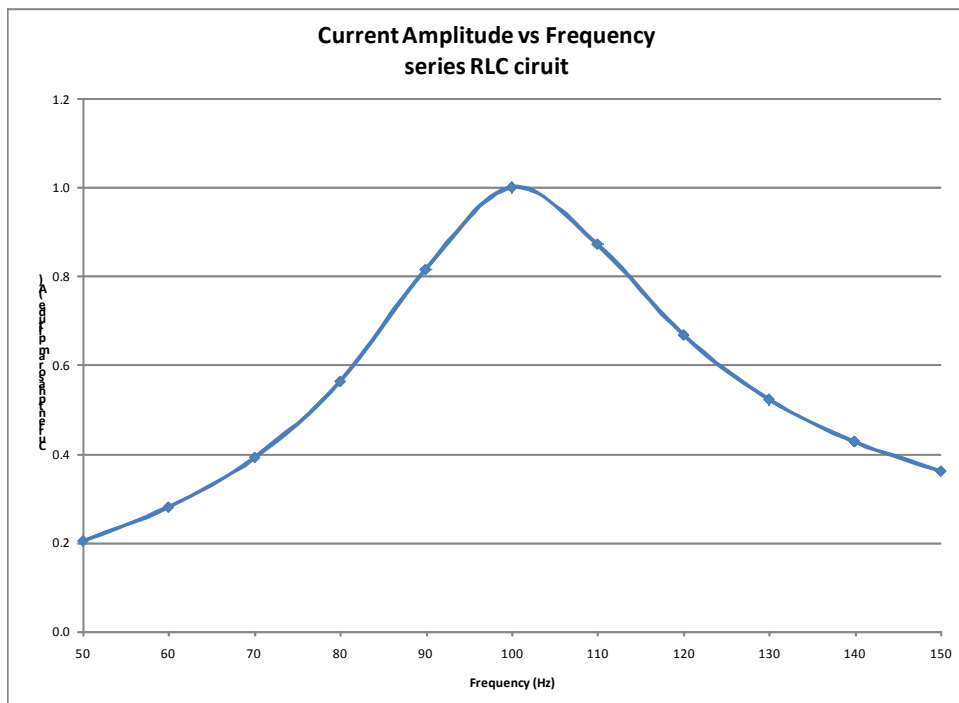
For convenient reference the current amplitude and phase are also provided as user-defined outputs.

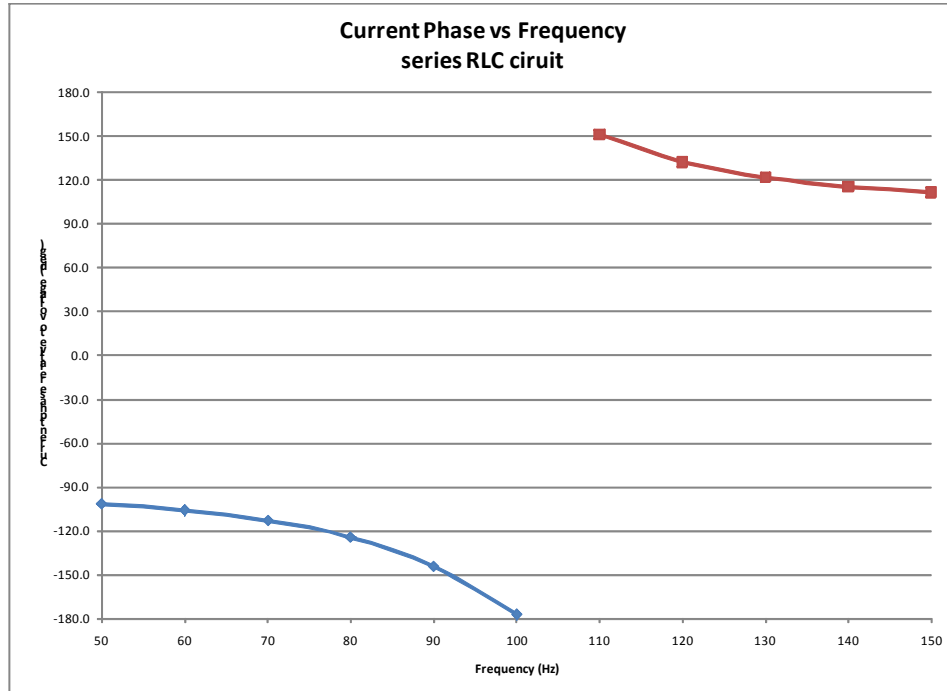
**Iamp** current amplitude 3.601E-01  
 IRamp

**Iphase** current phase 1.111E+02  
 IRphase

The defining values *IRamp* and *IRphase* are other user-defined variables exported from the *resistor* component. It doesn't matter which component since the currents are all the same in a series circuit.

The model is setup to map frequency over the range of 50 –150 Hz. When the resulting Sage.map file imported into MS Excel and plotted the results are the familiar amplitude and phase response curves for a damped resonant system:





## Theoretical Resonant Frequency

Equating the voltage drop across the voltage source to the voltage drops across the three components and representing the component voltage drops by their defining equations, the governing equation between voltage source and current in this circuit is

$$\Delta \dot{V}_{source} = R\dot{I} + L\ddot{I} + I/C$$

Where dots refer to time derivatives. Under the usual complex (phasor) analysis assumptions, namely that physical current is the real part of  $Ie^{i\omega t}$  and likewise for voltage, this equation becomes

$$I = \frac{\Delta V_{source}}{i(L\omega + 1/\omega C) + R}$$

It follows that the amplitude of current  $I$  is inversely proportional to the magnitude of the denominator on the right side. That magnitude is

$$|i(L\omega + 1/\omega C) + R| = \left( (L\omega + 1/\omega C)^2 + R^2 \right)^{1/2}$$

Differentiating with respect to  $\omega$  and setting the result equal to zero shows that the minimum magnitude in the denominator (maximum current amplitude) occurs at

$$\omega_0 = \sqrt{\frac{1}{LC}}$$

Converted to Hz this is just the above user variable FreqResonance, which works out to 100.7 Hz in this case, in good agreement with the above plot.

## Higher Harmonics

Something not considered in linear (phasor) resonant circuit analysis is the presence of higher harmonics in the voltage and current solutions. But no problem for the Sage model. You can investigate these harmonics by including higher order terms in the *voltage source* input  $F\Delta V$ .