Sage Model Notes

TAdemo.ptb

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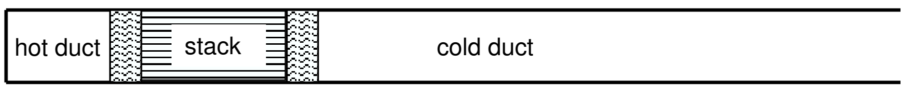
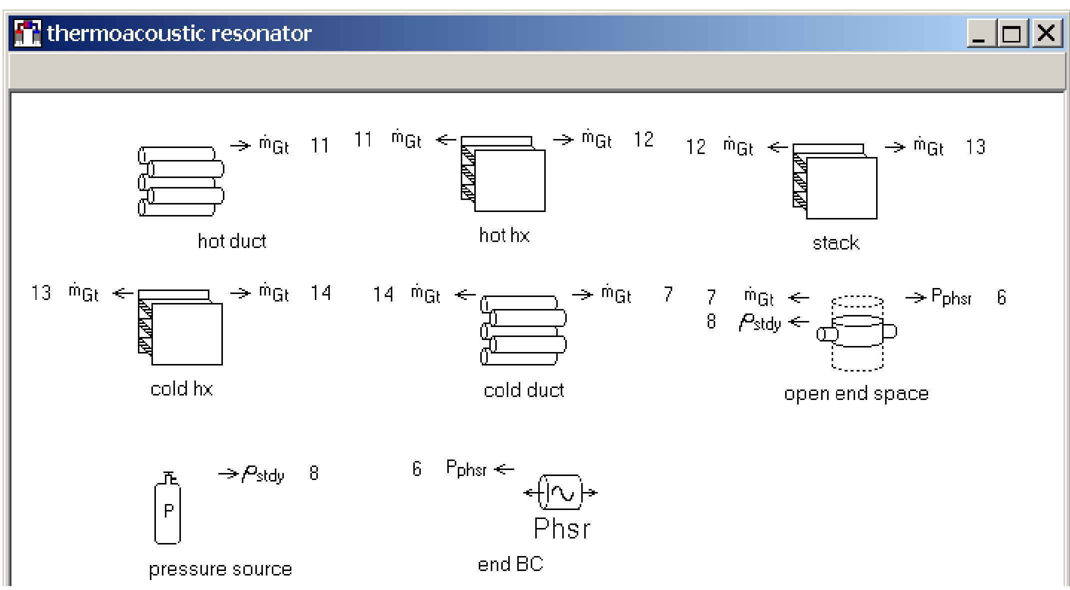
A model of the thermoacoustic demonstrator engine built by Greg Swift and associates and reported in [1]. Similar to a quarter-wave organ pipe, producing a loud sound through the thermoacoustic action of a stack of parallel plates near the closed end. Here is a schematic of the physical layout:



In the Sage model the components are arranged in order of occurrence with hot parts at the top of the edit window and cold parts at the bottom:



Components after the *cold duct* are fictitious (non-physical) components required to implement the acoustic boundary condition at the open end. The piston attached to the gas space actually drives the acoustics within the engine, but Sage's optimizer adjusts its amplitude so as to be consistent with the required acoustic impedance boundary condition of an open-ended tube. More on that below. The acoustical power output is just the mechanical power delivered to the piston once its motion is adjusted to satisfy the open-end impedance boundary condition. It is available in user-defined variable Wrad within the *end BC* piston.

The spacing between the plates in the *stack* is of the order of the thermal diffusion length in the gas, so accurate modeling requires the complex Nusselt number formulation. Therefore, the stack is modeled as rectangular channels, which employ the complex Nusselt number formulation, rather than as a foil matrix, which employs the real-valued low Valensi number limit.

Open End Impedance

The variable volume space at the *cold duct* open end represents the atmosphere immediately outside the duct. The volumetric displacement of the piston represents the displacement of the radiated sound to parts beyond. The phasing of this piston is arbitrary (the phasing of internal acoustics adjusting accordingly) but the amplitude must be set to satisfy the acoustic conditions at the tube end. (Actually, the piston phase is set to 25 degrees in order to offset the phase of the computational grid and ward off a slow-convergence problem that sometimes occurs when flow-reversal times coincide with nodes in the solution grid.) The operating frequency must also be set to match acoustic conditions, because a thermoacoustic resonator is a self-exciting device that determines its own frequency. So there are two variables to solve for, operating frequency and fictitious piston amplitude, subject to two conditions, which turn out to be formulated in terms of the acoustic impedance at the open tube end.

Morse and Ingard ([2], section 9.1, pp. 467–474) discuss the issue of impedance in acoustical ducts. Assuming the pressure and velocity at the desired point of interest in a plane standing are given by the complex formulation $\boldsymbol{p}e^{-i\omega t}$ and $\boldsymbol{u}e^{-i\omega t}$, the impedance is just the ratio of the complex amplitudes $\boldsymbol{p}/\boldsymbol{u}$. They put this in the form

$$\boldsymbol{p}/\boldsymbol{u} = \rho c(\theta - i\chi) \tag{1}$$

and then show some curves in figure 9.2, derived from [3], on the basis of which the following approximations appear reasonable at the open end of a tube of diameter D for a wave of wavelength λ

$$\theta \approx \frac{\pi^2}{4}(D/\lambda)^2 \quad (2)$$

$$\chi \approx 1.927 D/\lambda \quad (3)$$

These approximations hold only for $D/\lambda < 1/10$. As D/λ approaches zero, these impedance values serve to reflect most of the sound back into the tube with a pressure node just outside the tube end. Consequently, long narrow tubes radiate sound poorly.

As modeled by Sage, the acoustic impedance at the open end may be formulated in terms of the pressure and negative-boundary mass flow rate of the variable volume space to which the open tube end is attached. Complex pressure amplitude $\boldsymbol{p}$ is just $p_c + ip_s$ where p_c and p_s are the cosine and sine coefficients of the first harmonic of the FPMean Fourier series. Velocity amplitude $\boldsymbol{u}$ is $(1/\rho A)(\dot{m}_c + i\dot{m}_s)$ where $\dot{m}_c$ and $\dot{m}_s$ are the cosine and sine coefficients of the first harmonic of the FRhoUANeg Fourier series. A velocity Fourier series is not directly available. So the impedance at the open end is just

$$\boldsymbol{p}/\boldsymbol{u} = \frac{\rho A}{\dot{m}^2}\left[(p_c\dot{m}_c + p_s\dot{m}_s) - i(p_c\dot{m}_s - p_s\dot{m}_c)\right] \quad (4)$$

Equating the right sides of equations (1) and (4), substituting the above approximations for θ and χ, then $\pi D^2/4$ for A and c/f for λ and simplifying gives the final form of the open-end impedance constraints used in the model:

$$\frac{c}{f^2\dot{m}^2}(p_c\dot{m}_c + p_s\dot{m}_s) = \pi \quad (5)$$

$$\frac{D}{f\dot{m}^2}(p_c\dot{m}_s - p_s\dot{m}_c) = 2.45 \quad (6)$$

Optimization

User-defined variables in the *open end space* implement the above impedance constraints and Sage's optimizer solves the operating frequency (Freq) and piston amplitudes Xamp in order to satisfy the constraints.

References

[1] G. W. Swift, *Thermoacoustic Engines*, J. Acoust. Soc. Am., **84**(4), October 1988

[2] P.M. Morse & K. U. Ingard, Theoretical Acoustics, McGraw-Hill, (1968)

[3] H. Levine and J. Schwinger, *On the Radiation of Sound from an Unflanged Circular Pipe*, Phys. Rev. **73**:383, (1948)