Sage Model Notes

GasCompressorTimedValves.scfn

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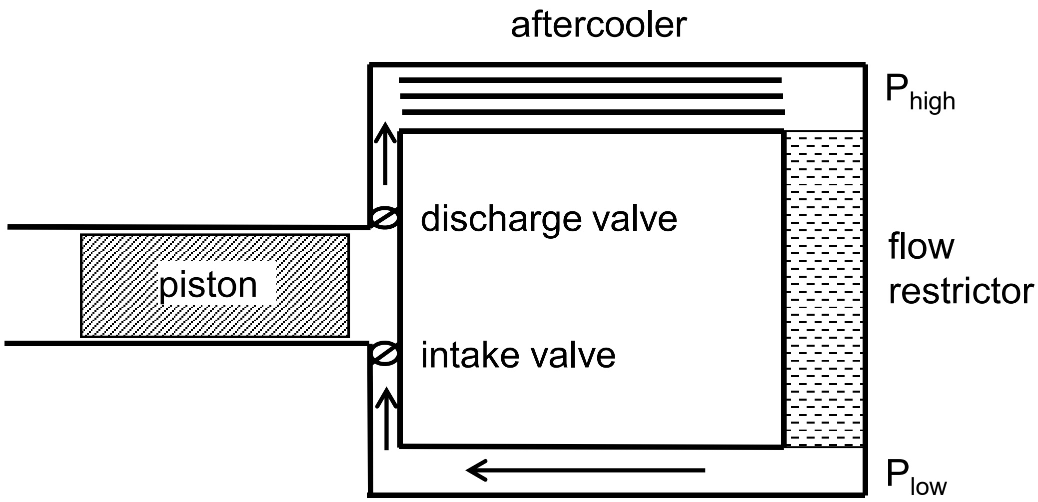
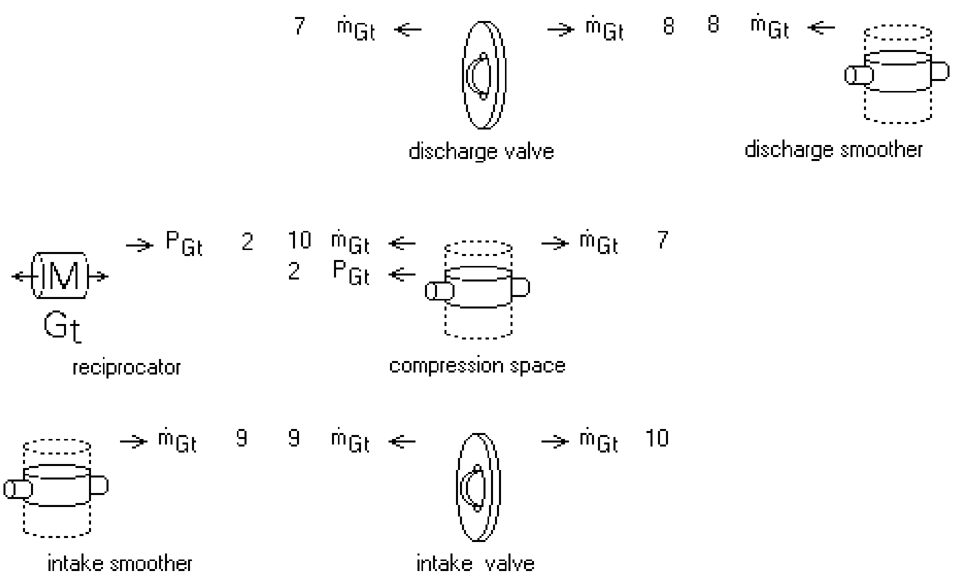
A piston-type gas compressor with time-dependent intake and discharge valves and a closed flow circulation loop:



This model is essentially the same as the gas compressor model with pressure-activated check valves (*GasCompressor.scfn*) except the intake and discharge valves are implemented as time-dependent valves where the opening and closing times are specified as functions of time:



In a typical compressor, the compression stroke begins just after piston bottom-dead-center (just after the intake valve closes) and the pressure increases from the intake to discharge pressure before the discharge valves opens. Otherwise there will be a backflow from the discharge line into the compression space. The discharge valve remains open until piston top-dead-center, when it closes. The expansion stroke begins just after piston top-dead-center, and the pressure decreases from the discharge to intake pressure before the intake valve opens, again preventing backflow through the valve. The intake valve remains open until piston bottom-dead-center, when it closes and the cycle repeats.

In this model the intake and discharge valves have square-wave open times similar to those of the model *GMSingleStageSqareWaveWalves.ptb*. As in that model, the valve timings are established by variables `OpenFrac` and `Phase`, which define the fraction of the cycle period the valve is open and the midpoint of the open time. Except in the present model `OpenFrac` and `Phase` are themselves variables. In the case of the intake valve:

```
OpenFrac         fraction cycle valve open              4.171E-01
  (180 - PhaseLow)/360
Phase            open time midpoint                     1.049E+02
  0.5*(PhaseLow + 180)
```

In the case of the discharge valve:

```
OpenFrac         fraction cycle valve open              2.262E-01
  (360 - PhaseHigh)/360
Phase            open time midpoint                     3.193E+02
  0.5*(PhaseHigh + 360)
```

Variables `PhaseLow` and `PhaseHigh` are the intake and discharge valve opening times that depend on the desired pressure ratio of the compressor, according to the theory below.

Mathematically the square-wave function defining the open time for a valve with `OpenFrac` = *b* and `Phase` = *α* is given by the Fourier cosine series with terms of the form

$$c_n \cos(n\omega t + r_n)$$

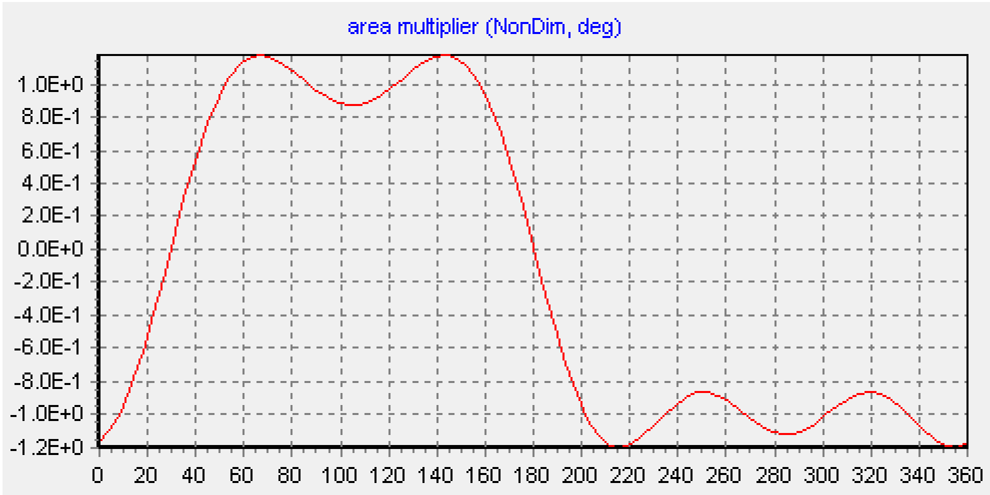
where

$$c_n = \frac{4}{n\pi}\sin(n\pi b)$$

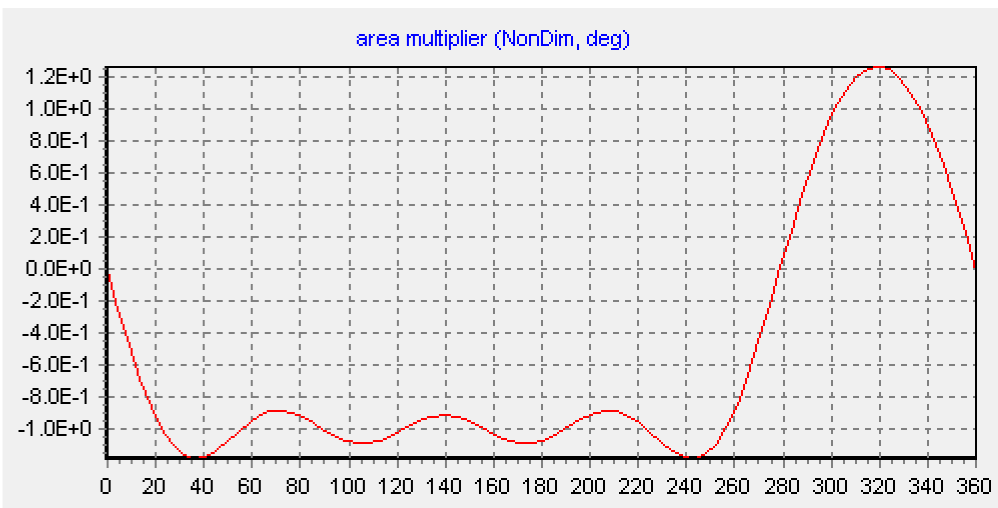
$$r_n = -n\alpha$$

Such square-wave functions are used to recast the `Frestrict` open-area multiplier inputs for the two valves. When viewed with the *View Interpolation* control of the recast input dialog the `Frestric` functions look the plots below.

Intake valve



Discharge valve.



The valves are open when the area muliplier is greater than 0 (actually when greater than the variable `MinRestrict`, which is 1.0E-3 in the current model).

Adiabatic Compression and Expansion

For this model the *piston constraine*r component drives the piston with the motion

```
FX                displacement (m, deg)                    0.000E+00...
  ( 4.500)E-03 Amp
  ( 0.000)E+00 Arg
```

Where the amplitude (4.5 mm) comes from the *Xamp* input of the root model. This means the piston is moving in the form

$$x = x_1 \cos \tau$$

where $\tau = \omega t$ is the cycle angle, and x_1 is the amplitude. Since a positive piston displacement decreases volume in the model, the volume variation is of the form

$$V = V_0 - V_1 \cos\tau$$

When both valves are closed it is reasonable to assume that an adiabatic P-V relationship holds in the compression space. The differential equation for an adiabatic pressure variation in terms of the above volume variation is

$$dP = -\gamma \frac{P}{V_0 - V_1 \cos\tau} dV$$

Which may be written in the form

$$\frac{dP}{d\tau} = -\gamma \frac{P}{V_0 - V_1 \cos\tau} \frac{dV}{d\tau} = -\gamma \frac{P \sin\tau}{V_0 / V_1 - \cos\tau}$$

There are two solutions of interest, the compression process starting at $P = P_{low}$ and $\tau = \pi$

$$P = P_{low} \left(\frac{V_0/V_1 + 1}{V_0/V_1 - \cos\tau} \right)^{\gamma} \tag{1}$$

and *the expansion process starting at $P = P_{high}$ and $\tau = 0$*

$$P = P_{high} \left(\frac{V_0/V_1 - 1}{V_0/V_1 - \cos\tau} \right)^{\gamma} \tag{2}$$

Substituting $P = P_{high}$ in equation (1) and solving for τ gives the ideal opening time for the discharge valve as

$$\tau_{high} = \arccos\left[\frac{V_0}{V_1} - \left(\frac{P_{low}}{P_{high}} \right)^{1/\gamma} \left(\frac{V_0}{V_1} + 1 \right) \right]$$

Substituting $P = P_{low}$ in equation (2) and solving for τ gives the ideal opening time for the intake valve as

$$\tau_{low} = \arccos\left[\frac{V_0}{V_1} - \left(\frac{P_{high}}{P_{low}} \right)^{\gamma} \left(\frac{V_0}{V_1} - 1 \right) \right]$$

These opening times are implemented in the model as variables variables `PhaseHigh` and `PhaseLow` of the *single-acting compressor* submodel.

```
PhaseLow         intake valve opening angle              2.986E+01
   (180/Pi) * ArcCos(Vr - Power(Phigh/Plow, 1/Gamma)*(Vr -1) )
PhaseHigh        discharge valve opening angle           2.786E+02
  360 - (180/Pi) * ArcCos(Vr - Power(Plow/Phigh, 1/Gamma)*(Vr +1) )
```

The definining expressions for PhaseLow and PhaseHigh are formulated in terms of variables defined in the *single-acting compressor* submodel:

```
Inputs
  Plow             target low pressure (Pa)                        1.000E+05
  Phigh            target high pressure (Pa)                       3.000E+05
Outputs
  Vr               V0/V1 volume ratio                              1.111E+00
    XpMax / XpAmp
  Gamma            adiabatic exponent                              1.398E+00
     Gas.Cp0(300) / (Gas.Cp0(300) - Gas.Rgas)
```

where `XpAmp, XpMax` and `Gas` are inputs of the root model.

Calibrating the return path

The model pumps nitrogen in a close loop with the return path implemented by the flow restrictor component, consisting of a tube filled with a random-fiber matrix. The flow resistance of the return path must be calibrated to the compressor so the pressure drop through the flow restrictor balances the pressure rise in the compressor. In the present model this is handled by the optimizer, according to this specification:

```
OPTIMIZED VARIABLES                     SUBJECT TO CONSTRAINTS

7.1 random fiber matrix
  Dfiber                                FPmean.Mean = Phigh
```

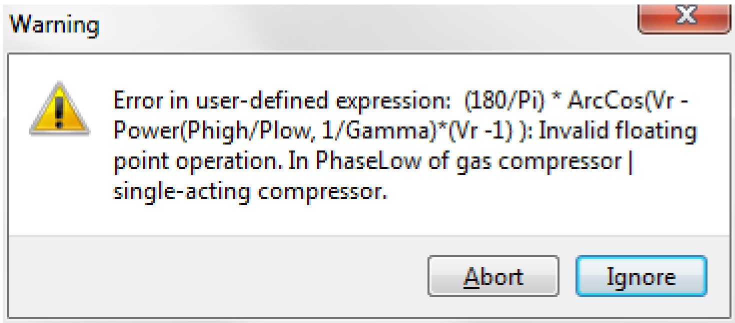
The optimizer adjusts the fiber diameter in the flow restrictor in order to make the mean pressure in the *discharge smoother* gas volume (just downsteam of the discharge valve) equal to the pressure Phigh. The target low and high pressures are defined in the compressor submodel.

```
  Plow             target low pressure (Pa)                        1.000E+05
  Phigh            target high pressure (Pa)                       3.000E+05
```

The low pressure is established by the charge pressure connection to the negative end of the flow restrictor gas volume.

Partial Cycle Exceptions

In an actual compressor, if the pressure ratio P_{high} / P_{low} is too high or there is too much dead volume in the compression space (`XpAmp << XpMax`) then it may be impossible for the compression stroke to increase the pressure to achieve the desired discharge pressure, in which case the compression space will just cycle through an adiabatic compression/expansion process without any net gas flow. In the Sage model this manifests itself in the form of exceptions raised during the calculation of the ArcCos functions in the expressions defining PhaseLow and PhaseHi. For example:



To fix this problem either decrease `Phigh` or increase `XpAmp`.

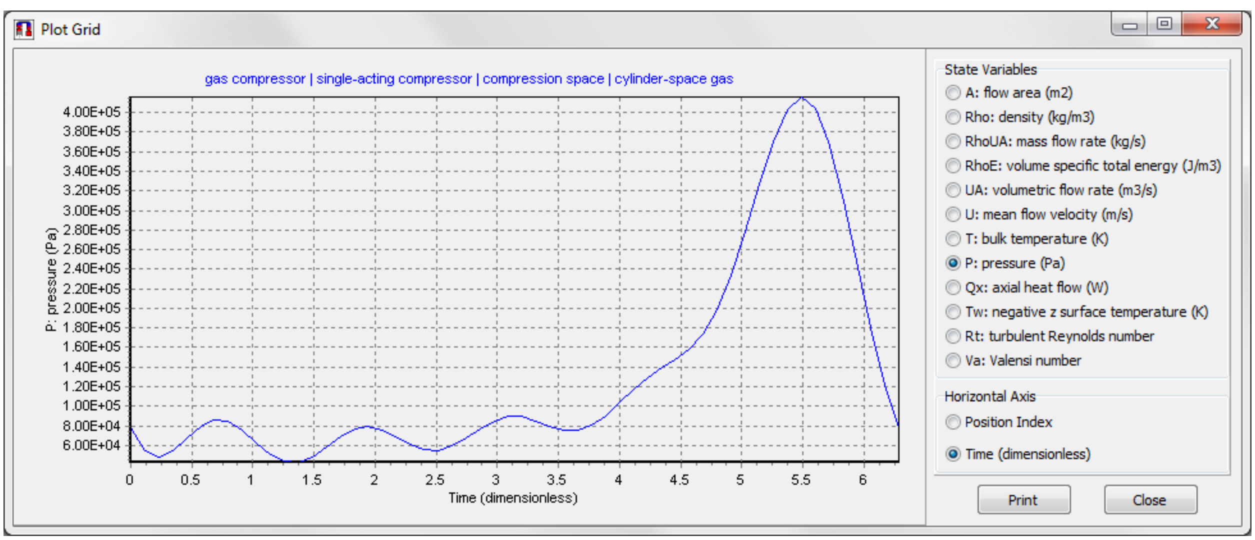
Time grid resolution

As the pressure ratio P_{high}/P_{low} increases the open time of the valves decreases and the pressure variation in the compression space becomes less sinusoidal. In order to properly resolve the compressor pressures and flow through the valves you may have to increase the number of time nodes in the computational grid, specified by root model input `NTnode`.

The present value of NTnode = 11 roughly suggests that the model can resolve valve open times on the order of 1/11 = 0.09 of the cycle period. This should be adequate for the `OpenFrac` values of 0.226 and 0.417 for the discharge and intake valves of the present model. In general, you should set NTnode at-least equal to the next odd number greater than the `1/OpenFrac` for the discharge valve.

Solution Plots

The Plot Solution Grid menu option (right-click context menu starting Sage v11) is very handy for understanding the compressor model and diagnosing problems. For example, here is a plot of the pressure in the compression space.



The pressure remains low when the intake valve is open (dimensionless time interval 0.52 to π) then increases rapidly until the discharge valve opens (dimensionless time 4.86). In theory the pressure should remain high and steady until the discharge valve closes (dimensionless time 2π) but apparently there is some pressure over-shooting going on, either because the grid cannot resolve the pressure in the short open time or there is a significant pressure drop across the discharge valve.

With NTnode = 11 there are 11 equal-spaced grid points defining the pressure in the above plot (12 if you consider the one at dimensionless time = 2π, which is equivalent to the point at 0, according to Sage's periodic time grid assumption). Those points tend to fall half-way between the peaks and valleys of the wiggles, which are Fourier-series interpolations of the solution between grid points. The fact that there are wiggles instead of smooth variations between grid points is an artifact of the plotting routine rather than a problem with the solution itself but it does suggest that there are higher harmonics in the solution that are not fully resolved by the current value of NTnode.

Also of interest are pressure plots in the intake and discharge smoothers and flow restrictor gas volumes. For example, here is the pressure distribution in the flow restrictor return path going from `Phigh` to `Plow` (flow is negative toward the left):