Sage Model Notes

InductorAirCore.scfn

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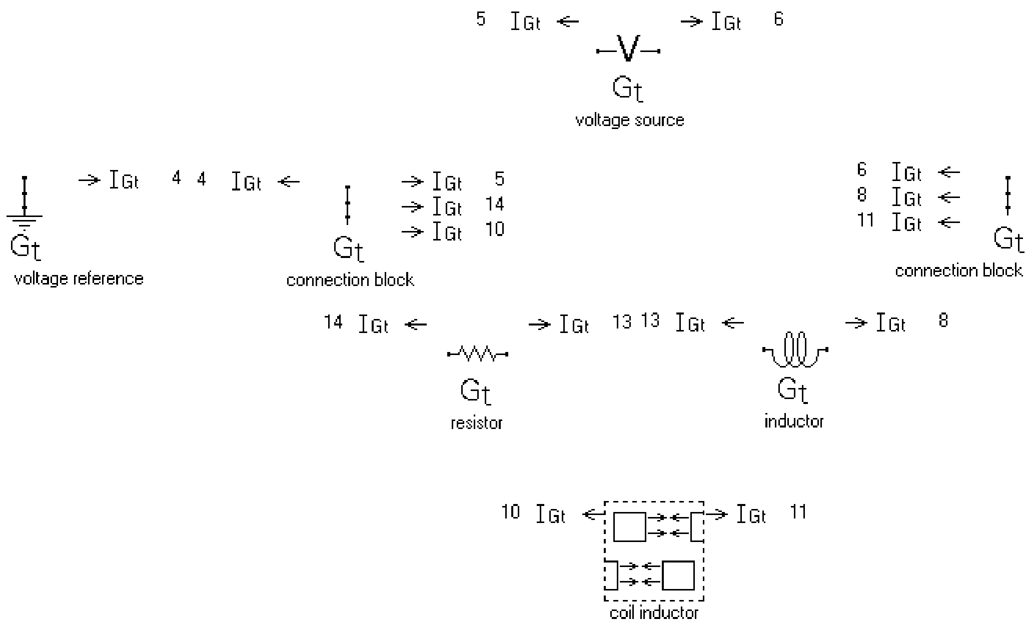
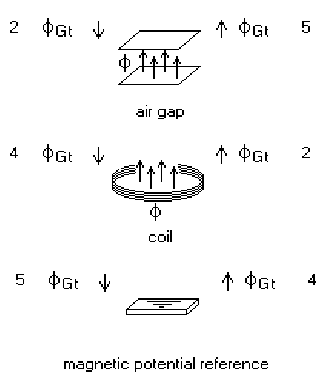
A model of an air-core solenoid from first principles including for comparison its equivalent RL (resistor-inductor) circuit element made from simple electrical components.

The Sage model looks like this:



A voltage source (top row) drives electrical current through two circuit elements in parallel. One circuit element (third row) is a simple resistor and inductor in series where the resistance and inductance are set as inputs. The other circuit element is a coil inductor submodel (bottom row) where the resistance and inductance come from coil and air-gap components. The contents of the coil inductor submodel are these:



The coil and air gap form a magnetic circuit anchored by a magnetic potential reference. Electrical current flowing through the coil produces a magnetic field that drives magnetic flux through the air gap. The changing magnetic flux induces a counter EMF in the coil that opposes the current change.

Solenoid Theory

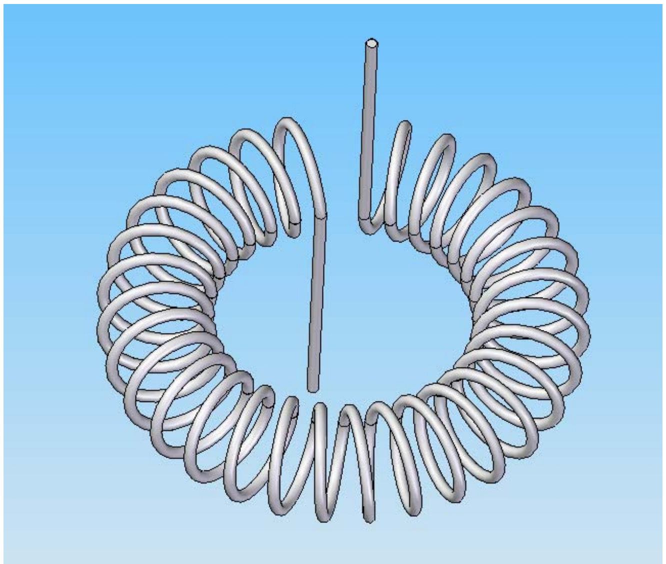
The submodel amounts to a solenoid, which is a coil of wire wound on a cylindrical form. J. D. Kraus tells us[1] that the inductance of a *long* solenoid is given by the formula

$$L = \frac{\mu N^2 A}{l}$$

Where

L	=	Inductance (H)
μ	=	Magnetic permeability of medium in core (H/m)
N	=	Number of wire turns (coil input Nturns)
A	=	Cross-sectional area of solenoid (air gap input Apath)
l	=	Length of solenoid (air gap input Lpath)

In practice the solenoid must be long compared the coil diameter so that magnetic flux leakage near the ends can be ignored. But in the Sage model there is no flux leakage. The length and core section area are defined by the air gap inputs *Lpath* and *Apath*. In the Sage model both ends of the air gap are connected to the same magnetic potential which essentially means that the air gap amounts to a closed torroidal loop. So if the Sage coil + air gap submodel were rendered faithfully as a solid model it might look something like this:



Coil Resistance

The Sage coil model includes some additional inputs besides N in the above formula:

Inputs

Dwire	wire diameter (m)	2.000E-03

[1] John D. Kraus, *Electromagnetics*, Fourth Edition, McGraw-Hill (1992), p. 235

Nturns	number turns in coil (NonDim)	1.000E+02
Dcentroid	coil centroid diameter (m)	1.000E-02
Alpha	coil packing factor (NonDim)	9.000E-01
Tcoil	coil temperature (K)	3.000E+02
Conductor	wire conductor material	copper

Many of these are practical inputs that relate to the space the coil occupies and the coil electrical resistance. The electrical resistance is an output that depends on the wire length, cross section area and resisistivity of copper at the specified temperature:

Rcoil	coil resistance (Ohm)	1.690E-02

The resistance input of the resistor in the parallel circuit element is set to the same value.

Another output of the coil model is the inductance, which the Sage manual tells us is an effective value calculated in a general way from coil voltage drop ΔV and electrical current rate of change dI/dt. Essentially the effective inductance is the component of ΔV in phase with dI/dt. When the two vary sinusoidally and are in phase the effective value is the same as the ideal value for a pure inductor. The effective inductance of the coil + air gap submodel is

Lcoil	effective inductance (H)	1.257E-04

For an air core the permeability is $\mu_0 =$ 1.257E-6 and the above inductance formula gives the same value as Sage, validating the Sage model.

The inductance of the inductor in the parallel circuit element is set to this same value. As a result the current flow in both circuit elements turns out to be the same, or nearly so (with 4 figure accuracy similar to resistance and inductance inputs), which further validates the Sage model.