Sage Model Notes

HeatExchanger-ConductiveMatrix.scfn

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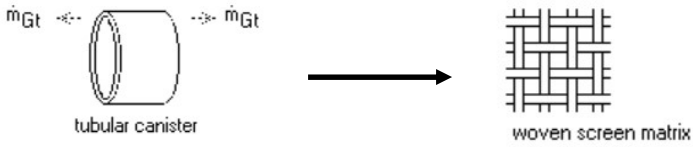
25 November 2024

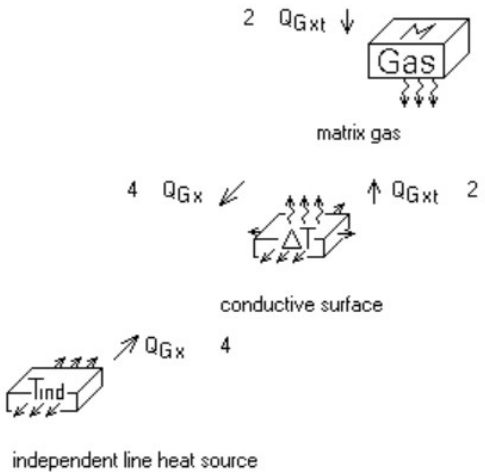
A variation of a porous-matrix heat exchanger where the matrix acts as a conduction path to the outer perimeter, which is held isothermal. See HeatExchanger-IsothermalMatrix.scfn for an option that eliminates the conduction path.

A typical physical implementation of this type of heat exchanger would be a stack of copper screens diffusion bonded to the internal surface of a copper container, providing an intermediate conduction path to an ultimate heat sink or source.

Heat exchangers are typically created from one of the duct-type geometries on the *Heat Exchangers* page of the root-level component palette. They can also be created starting from one of the canisters available on the *Canisters* page of the root-model component pallet, then dropping inside one of the porous matrix components available on the *Matrices* page inside the canister. This model implements a conductive woven-screen matrix heat exchanger within a tubular canister:



The canister connector arrows originate from the matrix gas component inside the random fiber matrix:



The matrix gas component is the only option in the *Gas Domain* page of the component palette inside random fiber matrix. There is a different type of matrix gas for each parent matrix type. Inside the duct gas component are positive and negative gas inlets that model the mass flow rate through the end boundaries of the matrix gas.

These come from the *Charge/Inlets* page of the component palette and are the source of the $\dot{m}_{Gt}$ arrows at the ends of the canister in the root model.

The independent line heat source components is an option on the *Matrix Solids* page of the component palette. It implements a time-constant boundary temperature distribution $T(x)$ according to independent cubic spline input `Tscr`. You might also consider using the line heater component to impose a constant heat flux boundary condition. It defines a useful output for keeping track of the total heat transfer through its connected boundary:

```
Qhx              total heat transfer                        0.000E+00
  QyPos
```

In this case the heat transfer is zero because there is no gas flowing through the heat exchanger. Qhx is defined in terms of QyPos, a built-in solution output. Re-defining it as a new variable helps keep track of things in large models where there may be many heat sources. For example, if you are implementing a heat-rejecting heat exchanger you might rename `Qhx` to `Qrej`.

The conductive surface component is another option on the *Matrix Solids* page of the component palette. It represents the matrix solid conduction path. Input `Solid` specifies the solid material and input `D` is the effective matrix conduction length. See sample model HeatExchanger-ThermalConductors for specific geometric examples of matrix conduction paths. It models steady thermal conduction (no variation with time) that varies with axial position x as solved on a computational grid.

The effective matrix conduction length is just $1/4$ the canister diameter for a cylindrical canister. Why? The exact solution governing temperature drop for radial heat flow in a porous circular cylinder with a uniformly distributed source or sink is: [1]

$$Q = 8\pi(1-\beta)kL\Delta T$$

where Q is the total heat flow passing through the outer boundary, $(1-\beta)k$ is the effective solid conductivity and L is cylinder length. The effective conductivity is written as the product of the solid fraction $1-\beta$ (β is porosity) and non-porous conductivity k in a crude attempt to account for the void part of the solid, which is presumed insulating. ΔT is the outer-boundary temperature minus the cross-section average temperature.

For the conductive-surface model, on the other hand, (see Sage User's Guide for details) the y directed temperature drop is governed by

$$Q = k\frac{(A_s/D)}{D/2}L\Delta T$$

where factor A_s/D is the solid z-thickness and $D/2$ is the y-directed conduction length. Equating the two Q's and canceling obvious factors gives

[1] J.P. Holman, *Heat Transfer*, Fourth Edition, McGraw-Hill (1976), example 2-3, p. 35

$$D^2 = \frac{A_s}{4\pi(1-\beta)}$$

But $A_s/(1-\beta)$ is just the total canister cross section area $\pi D_c^2/4$. Making this substitution gives

$$D = D_c/4$$

So all one has to do is set the effective conduction length D to $1/4$ the canister diameter and the y-directed temperature drop in the conductive surface will be a good approximation to the radial temperature drop in the porous solid.

Input `D` is automatically set to the parent canister inner diameter by recasting it as

```
D = 0.25 * Din
```

using the Specify | Recast Variables… menu item or Tools | Explore Custom Variables dialog.