Sage Model Notes

HeatExchangers-CounterflowRecuperative.scfn

D. Gedeon
2 December 2024

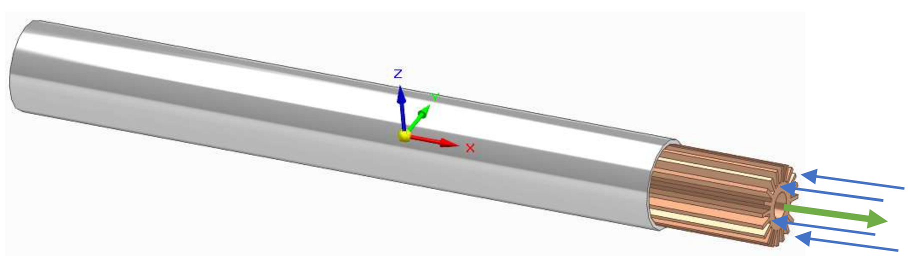
Other heat-exchanger sample models implement a single gas domain interacting with an isothermal surface or one or more thermal solids. The following samples show how to thermally connect two gas domains together using intermediary thermal solids.

These examples can form the basis for modeling recuperator counter-flow heat exchangers of the type used in a Rankine or Joule-Thomson thermodynamic cycle. The essence of such heat exchangers is the ability to transfer heat between two counter-flowing streams, recovering heat by cooling one stream and heating the other.

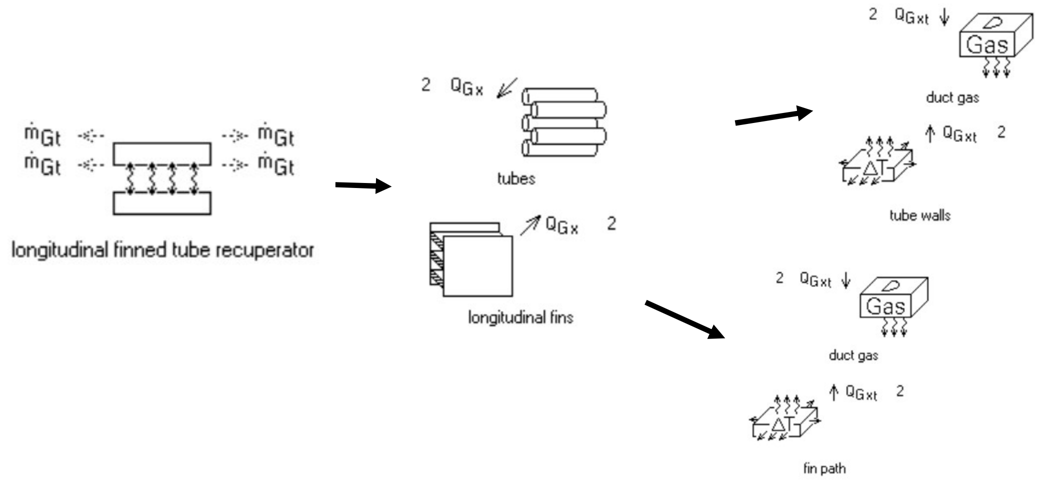
The conductive surfaces used to transfer heat filter out any AC component of heat transfer so that only the DC component is transmitted.

Longitudinal finned recuperator

This sample model corresponds to a bundle of tubes through which one fluid stream flows and a number of rectangular passages through which a counterflow fluid stream flows. The lengths of the tube and channels is the same. A typical example would be when longitudinal fins bonded to the outer tube walls form the boundaries of the counterflow passages.



In Sage, the model uses the *parallel container*, *tubular bundle, rectangular fins* and two instances of the *distributed conductor* components, except they are renamed *longitudinal finned tube recuperator*, *tubes*, *longitudinal fins*, *tube walls* and *fin path*:



The Q_{Gx} connectors between the *tubes* and *longitudinal fins* components are bumped up one level from the *tube walls* and *fin path* within.

To make the model easier to work with the top-level component contains a number of user-defined variables:

```
NfinnedTubes      number finned tubes (NonDim)            1.000E+00
NfinsPerTube      number fins per tube (NonDim)           2.000E+01
RfinBase          radius fin base (m)                     6.000E-03
Hfin              fin height (m)                          2.000E-03
```

The *tubes* component recasts two inputs consistent with the above inputs

```
Dtube = 2*(RfinBase - Twall)
Ntube = NfinnedTubes
```

The *longitudinal fins* component likewise recasts three inputs

```
Wchan = Pi*(2*RfinBase + Hfin) / NfinsPerTube - Tfin
Nchan = NfinnedTubes * NfinsPerTube
Hchan = Hfin
```

The *tube wall* recasts the fin conduction length to

```
D = Twall
```

And the *fin path* recasts the fin conduction length to

```
D = Hfin
```

The only non-obvious recast is for the fin effective channel width `Wchan`, which is defined so the cross-section area of the channels equals the void cross section of the fins. The combined channel cross section area of a single finned tube is

$$A\_c = n W H$$

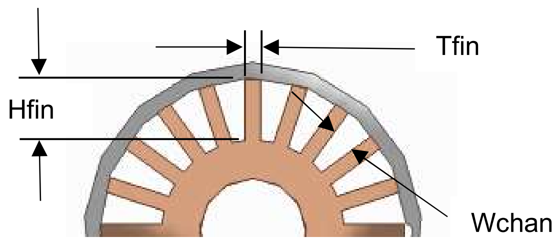
Where n is the number of fins per tube, W is the above `Wchan` and H is `Hchan`. The void cross section area of the finned annular region between the bounding circles between the fin bases and tips of radii R_b (above `RfinBase`) and R_t (`RfinBase + Hfin`) is

$$A_v = \pi(R_t^2 - R_b^2) - nt(R_t - R_b)$$

Where t is fin thickness (above `Tfin`). Equating A_c and A_v of the above two equation and solving for W simplifies to

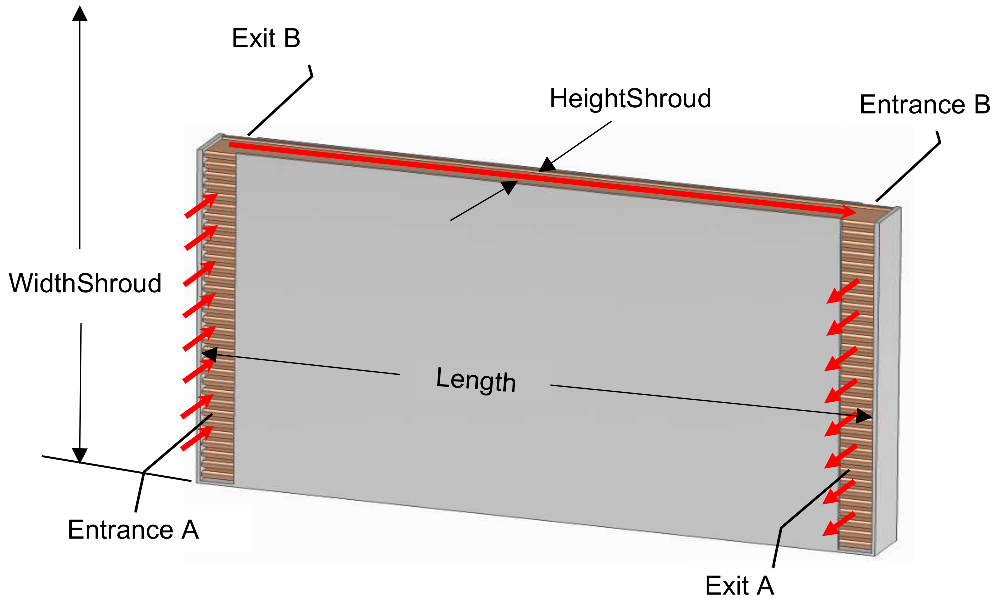
$$W = \frac{\pi(R_t + R_b)}{n} - t$$

The factor $\pi(R_t + R_b)$ on the right side is just the circumference of a circle passing through the fin mid-points. In other words, the effective channel width is just the average arc length of between the fins.

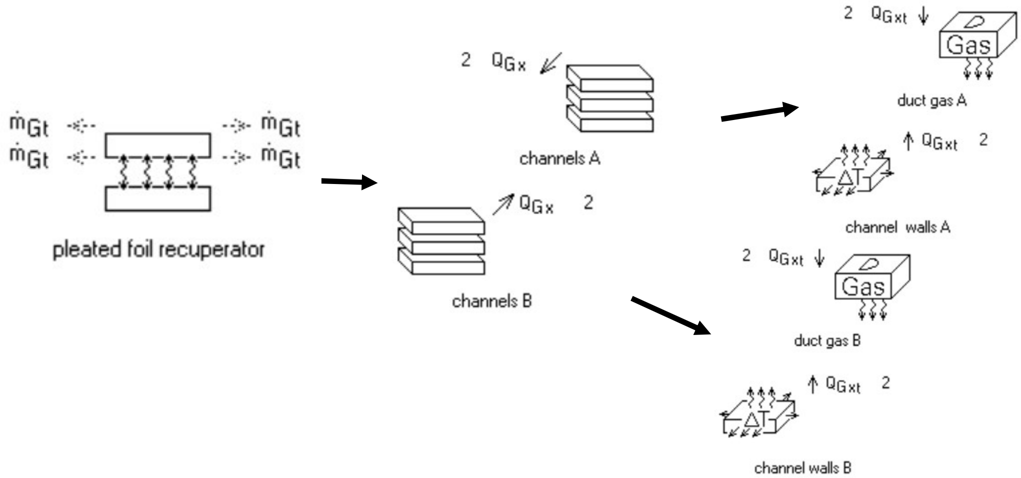


Pleated foil recuperator

This sample model corresponds to a sheet of formed foil-fin material sandwiched between two plates so-as to form alternating flow channels belonging to separate A and B counterflow fluid streams. The entrance and exits for the A stream are formed by rectangular entrances at opposite ends of one side. Likewise for the B stream on the other side, as in the sectioned view below of the lower half of such a recuperator.



This model might form the basis for an air preheater in a combustion system where both fluid streams are near atmospheric pressure and there is no structural loading on the flat plates.
.
In Sage, the model uses the *parallel container*, two instanced of *rectangular channels* and two instances of the *distributed conductor* components, except they are renamed *pleated foil recuperator*, *channels A, channels B*, *channel walls A* and *channel walls B*:

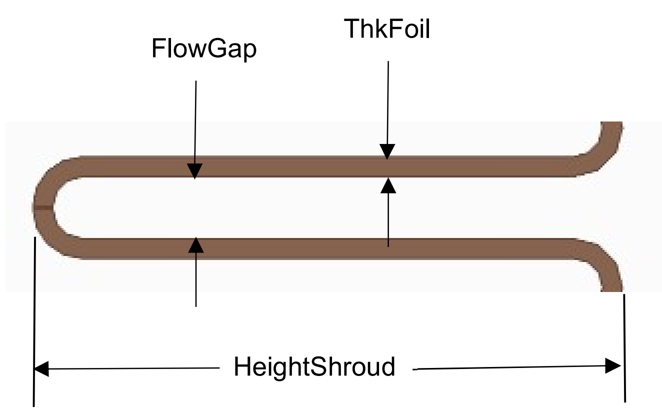


The Q_{Gx} connectors between the *channels A* and *B* components are bumped up one level from the *channel walls A* and *B* within.

To make the model easier to work with the top-level component contains a number of user-defined variables:

WidthShroud	shroud width (m)	2.400E-01
HeightShroud	shroud inside height (m)	1.500E-02
FlowGap	fluid flow passage (m)	1.500E-03
ThkFoil	foil thickness (m)	5.000E-04

The first two are illustrated on the above diagram. The next two are illustrated in the following rendering of a repetitive fin element:



There are also two user defined outputs of use below:

PleatSpacing	center-to-center spacing	2.000E-03
FlowGap + ThkFoil		
NchanPerSide	number channels each side	6.000E+01

```
0.5*WidthShroud / PleatSpacing
```

Channels A and *B* recasts their inputs consistent with the above variables

```
Twall = 0.5*ThkFoil
Wchan = FlowGap
Hchan = HeightShroud - ThkFoil
Nchan = NchanPerSide
```

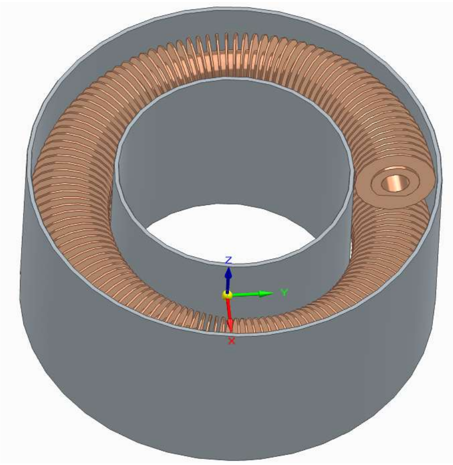
Since channels share a common wall is makes sense to define the wall thickness for each to be half the foil thickness. *Channels A* and *B* recasts the fin conduction length to

```
D = Twall
```

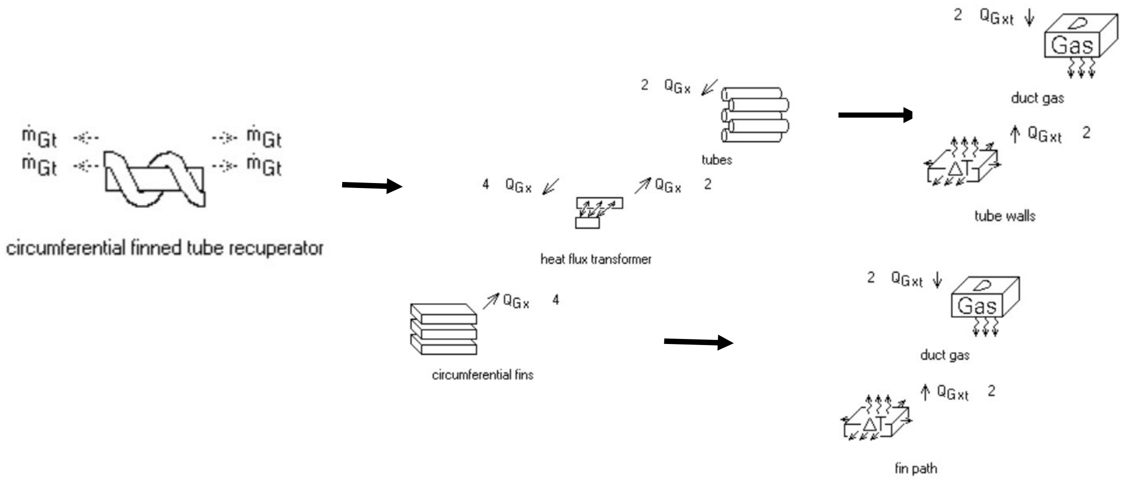
Another possible arrangement of this type of recuperator wraps the foil fins into a circular shape with cylindrical inner and outer walls replacing the flat plates. Adapting the sage model to this geometry requires only replacing the `WidthShroud` and `HeightShroud` inputs with appropriate equivalents for a circular geometry and updating some of the recasts on which they depend accordingly.

Circumferential finned recuperator

This sample model corresponds to a single tube wrapped circumferentially with fins of uniform height (i.e. circular envelope). The finned tube is helically wrapped within an annular shroud. One fluid stream flows through the tube and a counter flow stream between the fins within the annular shroud. This type of heat exchanger might be used in a Joule-Thomson cooling cycle were the high-pressure fluid flows through the tube and low-pressure fluid along the fins.



In Sage, the model uses the *multi-length container*, *tubular bundle, rectangular fins* and two instances of the *distributed conductor* components, except they are renamed *circumferential finned tube recuperator*, *tubes*, *circumferential fins*, *tube walls* and *fin path*:



The Q_{Gx} connectors between the *tubes* and *longitudinal fins* components are bumped up one level from the *tube walls* and *fin path* within. They are connected through the intermediate *heat flux transformer* component, necessary because the tubes and circumferential fins have different flow lengths. A heat-flux transformer knows nothing about the detailed geometry of the heat exchanger. It only transfers the DC component of heat flow uniformly between corresponding positions of the inter-connected components.

To make the model easier to work with the top-level component contains a number of user-defined variables:

```
ODtube          tube outer diameter (m)                  6.000E-03
ODfin           fin outer diameter (m)                   1.000E-02
ThkFins         fin thickness (m)                        2.500E-04
SpacingFins     fin to fin spacing (m)                   1.000E-03
DmeanShroud     annular shroud mean diameter (m)         4.000E-02
NturnsShroud    number turns tube in shroud (NonDim)     4.000E+00
```

The *tubes* component recasts two inputs consistent with the above inputs

```
Dtube = ODtube - 2*Twall
Length = Pi * DmeanShroud * NturnsShroud
```

The length, in other words, is the circumference of a single turn in the shroud time the number of turns.

The *circumferential fins* component recasts five inputs

```
Length = NturnsShroud * (Sqr(ODfin) - Sqr(ODtube)) / (4*Hchan)
Twall = ThkFins
Wchan = SpacingFins - ThkFins
Hchan = 0.5 * (ODfin - ODtube)
Nchan = Pi * DmeanShroud / SpacingFins
```

See below for explanations of these recasts.

The *tube wall* recasts the fin conduction length to

```
D = Twall
```

And the *fin path* recasts the fin conduction length to the radial fin height

```
D = 0.5 * (ODfin - ODtube)
```

Regarding the *circumferential fins* recast inputs, it helps to think of an equivalent rectangular channel geometry where the number of channels corresponds to the number of fins in a single turn of the helically coiled tube. Under that interpretation the number of channels n is

$$n = \pi D_s / s_f$$

where D_s is the shroud mean diameter (diameter of tube-center helix) and s_f is the fin spacing. The channel width W and height H (above `Wchan` and `Hchan`) are recast so W is the gap between fins and the flow area per channel $W\ H$ is the minimum flow area. The channel length L is defined so the volume of the channels in the Sage model equals the volume of the active heat-transfer region between the fins in the actual geometry (smaller than the total void volume of the shroud the fins are packaged in). In the Sage model the total channel volume is

$$\mathrm{V_c} = \mathrm{n\,W\,H\,L}$$

In the actual geometry the volume of the combined annular regions between the fins is the total number of fins (number n per turn times number of turns m_t) times the volume between fins

$$V_f = \mathrm{n} m_t W \frac{\pi\left(D_f^2 - D_t^2\right)}{4}$$

Where D_f is the fin OD (above `ODfin`) and D_t is the tube OD (above `ODtube`). Equating V_c and V_f of the above two equation and solving for L gives

$$L = m_t \frac{\pi\left(D_f^2 - D_t^2\right)}{4H}$$

Substituting $D_f - D_t$ for H simplifies to

$$\mathrm{L} = m_t \frac{\pi}{4}\left(D_f + D_t\right)$$

Since the Sage geometry matches both the active volume between the fins and the gap it also matches the active fin surface area. To see this compare the wetted surface area per channel in the sage model $S_c = 2HL$ with the actual per-fin surface area $S_f = 2m_t\pi\left(D_f^2 - D_t^2\right)/4$. Substituting the first of the above equations for L into the equation for S_c shows the two are the same.

This circumferential-fin model as implemented above assumes the finned-tubes completely fill the annular shroud they are packaged in. In other words, the volume between the circular fins in the helical winding and at the ends of the shroud due to partial turns is not included in the model. That is fine for a steady-flow model where extra adiabatic volume is irrelevant. But it may not be acceptable in an AC-flow model where the time-varying pressure of the overall model depends on heat exchanger volume.