Sage Model Notes

HeatExchangers-SimpleIsothermal.scfn

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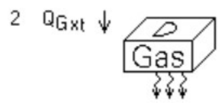
Isothermal Ducts

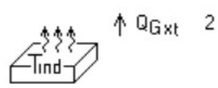
An isothermal duct-type heat exchanger comprises a container representing a particular heat exchanger geometry with a gas domain inside anchored by a time-constant temperature boundary surface. This model illustrates an implementation within a tube-bundle geometry:

isothermal tube bundle

The tube bundle is one of the options on the *Heat Exchangers* page of the component palette. It is also appropriate for drilled holes. The connector arrows originate from the duct gas components inside:

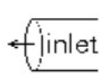


duct gas

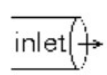


independent isothermal surface

The duct gas component is the only option in the *Gas Domain* page of the component palette inside. There is a different type of duct gas for each parent heat exchanger type. Inside the duct gas component are positive and negative gas inlets that model the mass flow rate through the end boundaries of the duct gas.



negative gas inlet



positive gas inlet

These come from the *Charge/Inlets* page of the component palette and are the source of the $\dot{m}_{Gt}$ arrows at the ends of the tube bundles in the root model.

The independent isothermal surface is an option on the *Duct Walls* page of the component palette. It is a relatively recent addition under Sage version 13 that sets its temperature distribution $T(x)$ from an independent cubic-spline input

```
Tsrc              surface temperature distribution (NonDim unit spline...
  (0.000E+00, 3.000E+02)
```

```
(1.000E+00, 3.000E+02)
```

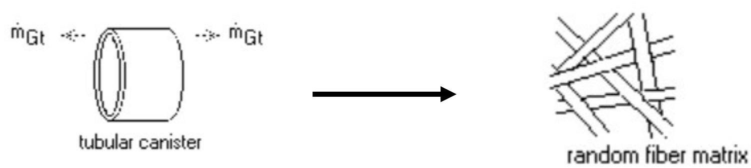
As such, the individual components of `Tscr` can be recast as dependent variables, mapped or optimized (menu items Specify **|** Recast Variables…, Specify **|** Mapped Variables…, or Specify **|** Optimized Variables…).

This is extremely useful for managing large and complicated models. For example you can change the temperature of several lower-level heat exchangers by changing a single root level user-defined input.

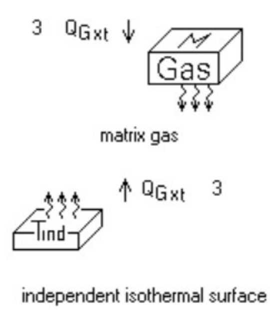
The isothermal surface is another option on the *Duct Walls* page of the component palette. Until recently it was the only isothermal surface option. It gets its temperature distribution $T(x)$ from the cubic-spline input `Tinit` of the parent heat exchanger component. This is convenient but suffers from some significant disadvantages. Namely, `Tinit` cannot be recast as a dependent variable in terms of user-defined inputs or other variables. Nor can it be mapped or optimized. These limitations arise because `Tinit` is defined as a constant, used for initializing and normalizing solved variables. You can still use this component if you want but it is no longer recommended.

Isothermal Matrix

Heat exchangers are typically created from one of the duct-type geometries on the *Heat Exchangers* page of the root-level component palette. They can also be created starting from one of the canisters available on the *Canisters* page of the root-model component pallet, then dropping inside one of the porous matrix components available on the *Matrices* page inside the canister. This model implements an isothermal random-fiber matrix heat exchanger within a tubular canister:



The canister connector arrows originate from the matrix gas component inside the random fiber matrix:



The matrix gas component is the only option in the *Gas Domain* page of the component palette inside random fiber matrix. There is a different type of matrix gas for each parent matrix type. Inside the duct gas component are positive and negative gas inlets that model the mass flow rate through the end boundaries of the matrix gas.

negative gas inlet   positive gas inlet

These come from the *Charge/Inlets* page of the component palette and are the source of the $\dot{m}_{Gt}$ arrows at the ends of the canister in the root model.

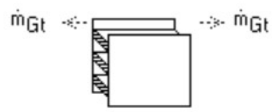
The isothermal surface component is one of the options on the *Matrix Solids* page of the component palette. See above discussion in in the Isothermal Ducts section. You can also use the surface heater component.

Physically speaking the matrix in an isothermal-matrix heat exchanger might represent something like a solar-radiation absorber, a radioisotope heat source or a matrix heated by electro-magnetic induction. Even without a physical interpretation isothermal matrix heat exchangers are often convenient abstract model components when there is a need to effectively heat or cool a gas stream in a model.

Matrix heat exchangers heated or cooled by solid conduction to the outer boundaries of matric can also be modeled using the thermal conduction path options available in the the *Matrix Solids* page of the component palette. See for example the sample model documented in HeatExchanger-ConductiveMatrix.docx.

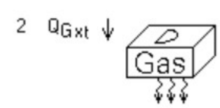
Isothermal Rectangular Fins

This sample model is based on the rectangular fins component on the *Heat Exchangers* page of the root-model component palette:

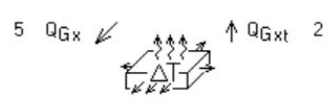


rectangular fins

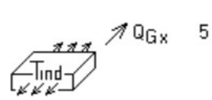
The connector arrows originate from the duct gas components inside:



duct gas



conductive surface



independent line heat source

The duct gas component is the only option in the *Gas Domain* page of the component palette inside. There is a different type of duct gas for each parent heat exchanger type.

Inside the duct gas component are positive and negative gas inlets that model the mass flow rate through the end boundaries of the regenerator gas.

negative gas inlet   positive gas inlet

These come from the *Charge/Inlets* page of the component palette and are the source of the $\dot{m}_{Gt}$ arrows at the ends of the tube bundles in the root model.

The independent line heat source and conductive surface are two of the options on the *Duct Walls* page of the component palette.

The independent line heat source provides a branching heat flow $Q(x)$ to the conductive surface component, consistent with the isothermal (time-constant) temperature distribution $T(x)$defined by its cubic-spline input

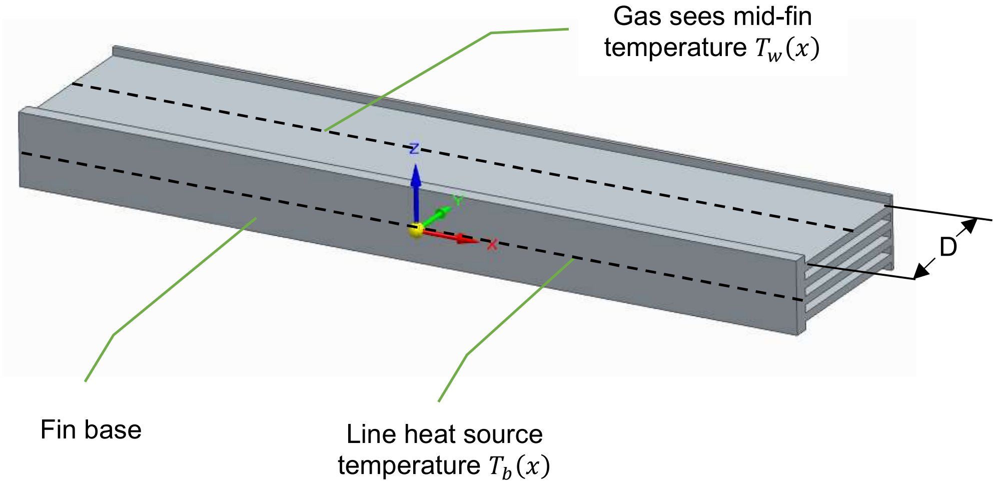
```
Tsrc              surface temperature distribution (NonDim unit spline...
  (0.000E+00, 3.000E+02)
  (1.000E+00, 3.000E+02)
```

The individual components of `Tscr` can be recast as dependent variables, mapped or optimized (menu items Specify | Recast Variables…, Specify | Mapped Variables…, or Specify | Optimized Variables…).

The conductive surface represents the metallic conduction path of the rectangular fins. See the HeatExchanger-ThermalConductors sample model for recommendations on modeling other thermal conduction path geometries.

Sage models the heat flow in the fin conduction path between the line heat source and the duct gas using a lumped-parameter approximation, which is the best the one-dimension Sage solution grid can do. See the Conductive Surface section of the Sage User's Guide (Help | PDF Manual) for details.

In the illustration below the fin base temperature is anchored by the line heat source temperature $T_b(x)$. Input `D` represents the total fin height. The duct gas sees the mid-fin temperature $T_w(x)$ a distance $D/2$ from the base.



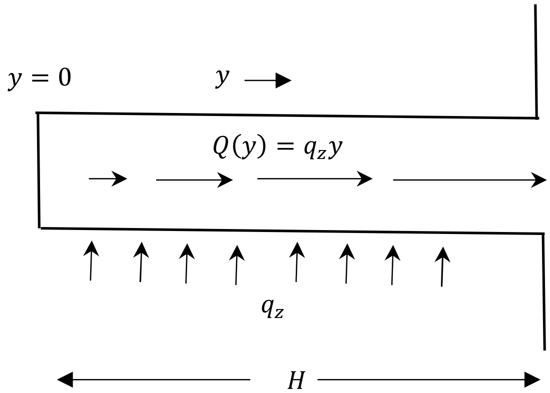
The temperature difference $T_b(x)$– $T_w(x)$ is determined by the total heat flow through a conduction path of length $D/2$ and total fin cross-section area.

Input `D` is automatically set to the parent rectangular fin height by recasting it as

```
D = Hchan
```

using the Specify | Recast Variables… menu item or Tools | Explore Custom Variables dialog.

Why is that recast appropriate? Consider the exact solution governing heat flow along a rectangular fin of height H subject to uniform heat flux q_z on the faces exposed to the gas, according to the diagram below.



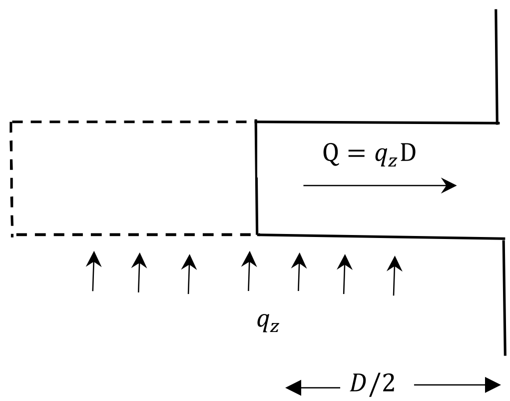
The total heat flow at the fin base is $Q(H) = q_z H$. The temperature variation along the fin can be expressed in terms of fin solid area A_s and thermal conductivity k as

$$\frac{dT}{dy} = \frac{Q}{kA_s} = \frac{q_z y}{kA_s}$$

So the total temperature drop along the fin is

$$\Delta T = \frac{q_z}{kA_s}\int_0^H y\,dy = \frac{q_z H^2}{2kA_s}$$

On the other hand, for Sage's conductive surface model the fin temperature drop is based on the entire heat flow $Q = q_z D$ for a fin of height D flowing over distance $D/2$, according to this diagram: (see Sage User's Guide for details)



The temperature variation along the fin is now uniformly (does not depend on y)

$$\frac{dT}{dy} = \frac{q_z D}{kA_s}$$

Producing a total temperature drop over distance $D/2$ of

$$\Delta T = \frac{q_z D^2}{2kA_s}$$

By inspection of the above two equations for ΔT it is clear that to produce the same ΔT for the same heat flux q_z requires that $D = H$.