

# Sage Model Notes

## HeatExchangers-ThermalConductors.scfn

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Some of the other heat-exchanger sample models used the *conductive surface* and *distributed conductor* components for modeling solid thermal conduction paths. Applying these samples to actual heat exchanger geometries has often been a point of confusion because it is not always clear which dimensions in the actual geometry correspond to the inputs of the Sage model.

This model attempts to shed more light on the matter by implementing a number of specific heat exchanger geometries with appropriate dimensions for Sage components recast in terms of user-defined inputs pertaining to the specific geometry. The individual root-level components of this model may be copy-and-pasted into your model as needed.

### Guidelines

First, keep in mind that you do not always need to use thermal conductors within heat exchangers. You can just anchor the gas domain within the heat exchanger directly to a line heat source, as many of the heat-exchanger sample models do, which then provides an isothermal surface boundary condition at the specified temperature. This approach effectively puts any parts of the heat exchanger beyond the wetted gas surface outside the scope of the Sage model.

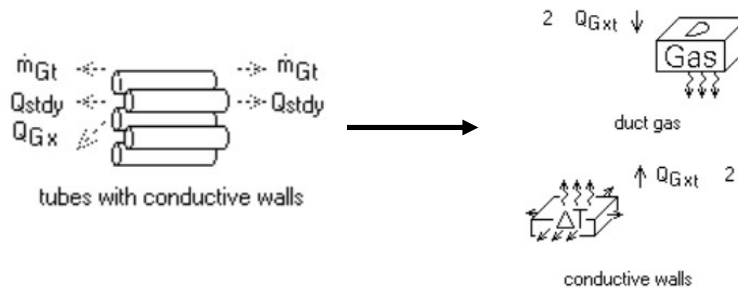
When you do need to model solid conduction paths think of them this way: The *conductive surface* within a heat exchanger models the solid material whose surface exchanges heat directly with the gas within the heat exchanger while the *distributed conductor* allows you to model a secondary solid conduction path representing a container wall or other structure outside the primary heat exchanger material. Both components have an input  $\Delta$  which specifies the distance along the direction of heat flow through the solid cross section, normal to the gas flow direction.  $\Delta$  only matters in the

event the distributed conductor is thermally connected along its length. If only connected at the ends then the solid cross-section area by itself governs heat flow, independent of  $D$ . When possible, both components calculate their cross-section area based on information inherited from their parent component. The exception is a stand-alone distributed conductor used for a secondary conduction path which requires a second input  $w$  and calculates cross-section area as the product  $w \times D$ . You can find a detailed discussion of Sage's thermal conductor components in the Sage User's Guide (Help | PDF Manual), chapter *Thermal Conductors In Heat Exchangers*.

## Tubes with Conductive Walls

This example models a heat exchanger comprising a number of tubes where you want to include the heat transfer (and temperature drop) through the tube walls to an external medium or along the tube walls (in the flow direction) to a source or sink at either end. They are one step more complicated than isothermal heat exchanger walls. The model comes with  $Q_{stdy}$  heat flow connections at each end and a single negative-directed  $Q_{Gx}$  heat flow connector. You might, for example, connect the  $Q_{Gx}$  connector (representing collective tube walls) to an isothermal line heat source to represent the temperature of a secondary heat transfer fluid. You might connect one or both of the  $Q_{stdy}$  connectors to a point heat source in order to model insulated tube walls with heat conduction along the tube walls in the gas flow direction. You can delete any connectors you don't want to use.

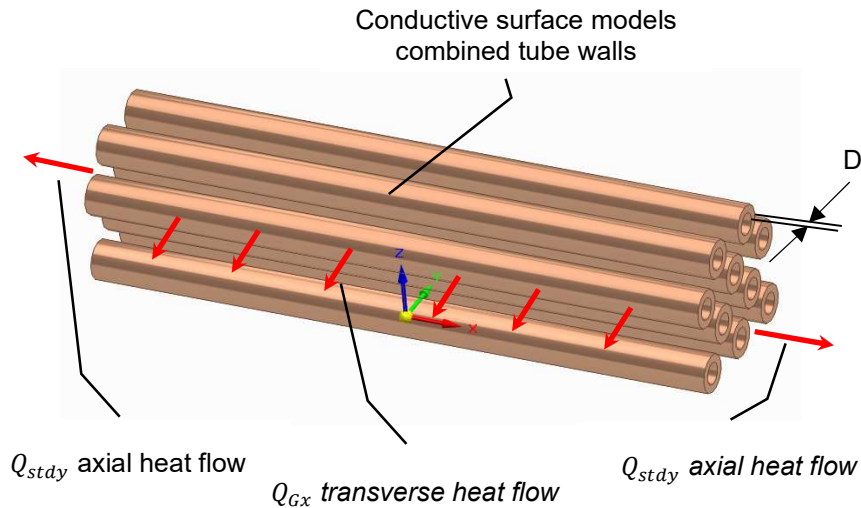
In the Sage model, the abstract rendering looks like this:



According to the actual geometry, the model renames the *tube bundle* component to *tubes with conductive walls* and the *conductive surface* component to *conductive walls*: The model recasts the  $D$  input of the *conductive walls* to the parent wall thickness input:

$$D = T_{wall}$$

The physical heat exchanger looks something like this:



The  $Q_{std}$  connections can be used to transfer heat through the tube ends, as if there were physical heat sink or source structures attached to the tube ends. The  $Q_{Gx}$  connection can be used to transfer heat through all the tube walls simultaneously. One option is to attach the  $Q_{Gx}$  connector to a line heat source or line heater component with temperature or heat flux adjusted based on a separate analysis of external heat transfer. Another option is to attach it to an opposite  $Q_{Gx}$  connector of a distributed conductor within another heat exchanger, although this can only be done in the context of a *parallel container* or *multi-length container* submodel. See the sample model HeatExchangers-CounterflowRecuperative.scfn or the chapter Submodels and Containers in the Sage User's Guide (Help | PDF Manual).

**Other Tube Geometries** You can create similar heat exchangers by starting with the rectangular channels or rectangular fins on the Heat Exchangers page of the root-level components palette in place of the tube bundle. The lower-level model components would be the same, apart from optional renaming of components and revising the recast expression for the distributed conductor  $D$  input to represent the appropriate conduction distance through the solid walls. For example, see sample model HeatExchangers-SimpleIsothermal.scfn.

## Drilled Hole Heat Exchangers

Imagine individual tubes with combined wall cross-sections swaged together into a single contiguous solid conduction path. That is the idea behind using Sage's tube-bundle component to model drilled holes in a block of solid conductive material. The parent tube component calculates a solid cross section area according to diameter, wall thickness and tube number inputs. You need to specify wall thickness to make this total solid cross section (heat exchanger output  $A_{sec}$ ) come out right. Either by hand or automatically by recasting input  $T_{wall}$  in terms of actual solid area,  $A_m$ , which to equal the solid area calculated by Sage must satisfy

$$A_m = n \frac{\pi}{4} ((d_i + 2t_w)^2 - d_i^2)$$

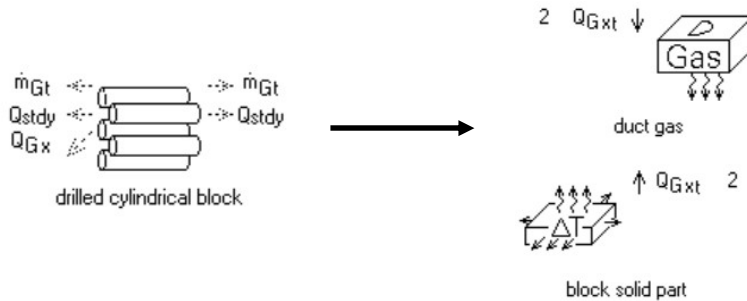
where  $n$  is tube number,  $d_i$  tube inner diameter and  $t_w$  wall thickness. Solving this for wall thickness requires that

$$t_w = \frac{1}{2} \left( \sqrt{d_i^2 + \frac{4A_m}{n\pi}} - d_i \right)$$

The examples below illustrate several possible arrangements of drilled-hole heat exchangers.

## Drilled cylindrical block

In Sage, the outward appearance of this model is the same as for the above model for tubes with conductive walls. Except the key components are renamed:



Other differences are the user-defined variables of the drilled cylindrical block, according to the above guidelines

```

Input
  Rblock          block radius (m)          1.000E-02

Output
  Am              solid cross section area   3.362E-04
  Pi * Sqr(Rblock) - Aflow

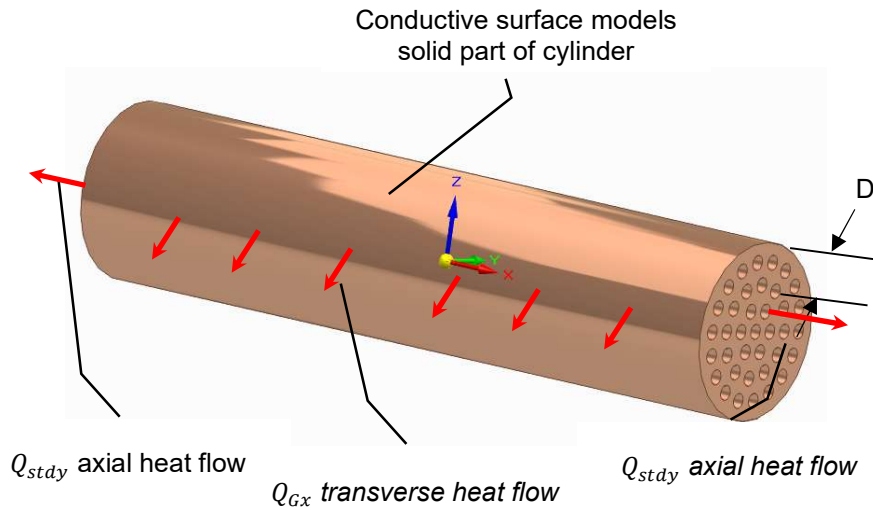
Recast
  Twall = 0.5*(Sqrt(Sqr(Dtube) + 4*Am/(Ntube * Pi)) - Dtube)
  
```

Input D of the block solid part is recast as

$$D = 0.5 * Rblock$$

See the documentation for sample model HeatExchanger-ConductiveMatrix.scfn for why D should be half the radius.

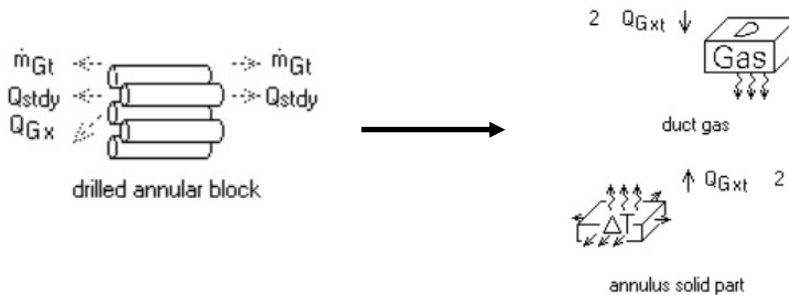
The physical heat exchanger looks something like this:



The  $Q_{stdy}$  connections can be used to transfer heat through the block ends. The  $Q_{Gx}$  connection can be used to transfer heat through the outer cylindrical surface. One option is to attach the  $Q_{Gx}$  connector to a line heat source or line heater component with temperature or heat flux adjusted based on a separate analysis of external heat transfer. See below for other options.

### Drilled annular block

In Sage, the outward appearance of this model is the same as for the above Drilled Cylindrical Block. Except the key components are renamed:



The user-defined variables of the drilled annular block are now

```

Inputs
  Rinner      annulus inner radius (m)      7.000E-03
  Router      annulus outer radius (m)     1.400E-02

Output
  Am          solid cross section area      3.299E-04
  Pi * (Sqr(Router) - Sqr(Rinner)) - Aflow

Recast
  Twall = 0.5*(Sqrt(Sqr(Dtube) + 4*Am/(Ntube * Pi)) - Dtube)

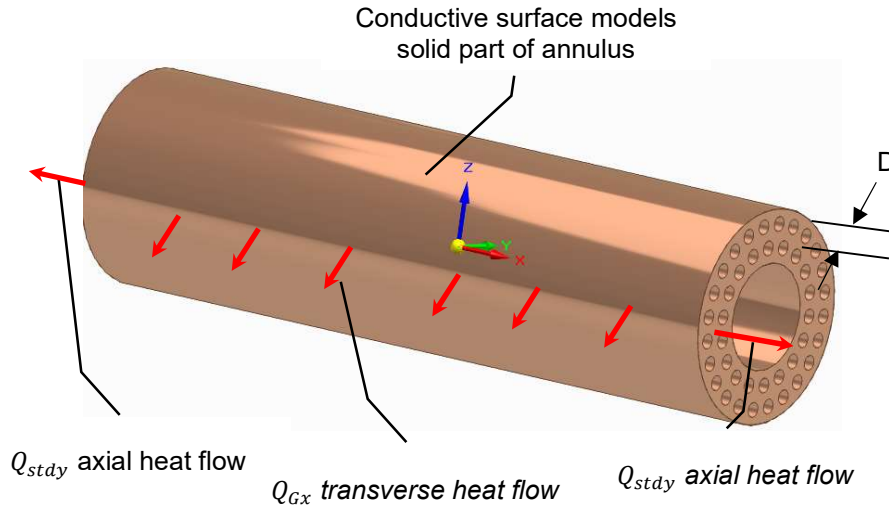
```

Input D of the annulus solid part is recast as the mean radial conduction path length

$$D = R_{outer} - R_{inner}$$

This amounts to approximating the radial heat flow by linear heat flow (see discussion in sample model HeatExchangers-SimpleIsothermal.scfn, Rectangular Fins section).

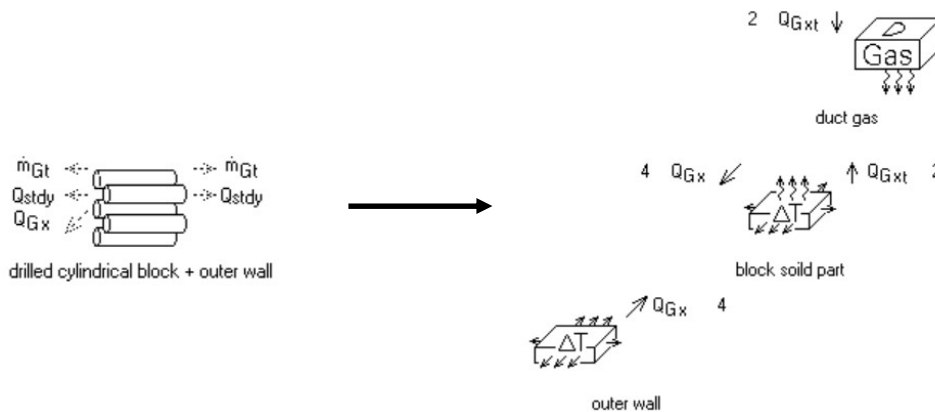
The physical heat exchanger looks something like this:



The  $Q_{stdy}$  connections can be used to transfer heat through the annulus ends. The  $Q_{Gx}$  connection can be used to transfer heat through the outer cylindrical surface. One option is to attach the  $Q_{Gx}$  connector to a line heat source or line heater component with temperature or heat flux adjusted based on a separate analysis of external heat transfer. See below for other options.

### Drilled cylindrical block bonded to conductive outer wall

This model extends the above Drilled Cylindrical Block model by connecting it to an outer wall which serves as the ultimate connection to a heat source or sink. The appearance in Sage is similar to the above Drilled Cylindrical Block except the key components are renamed and there is an additional distributed conductor component named Outer Wall:



The user-defined variables of the top-level component are

```
Inputs
  Rblock      block radius (m)          1.200E-02
  ThkOuterWall  outer wall thickness (m)  2.000E-03

Output
  Am          block solid cross section area  not compiled
  Pi * Sqr(Rblock) - Aflow

Recast
  Twall = 0.5*(Sqrt(Sqr(Dtube) + 4*Am/(Ntube * Pi)) - Dtube)
```

Input D of the annulus solid part is recast as for the stand-alone Drilled Cylindrical Block.

$$D = 0.5 * Rblock$$

Inputs W and D of the outer wall are recast as

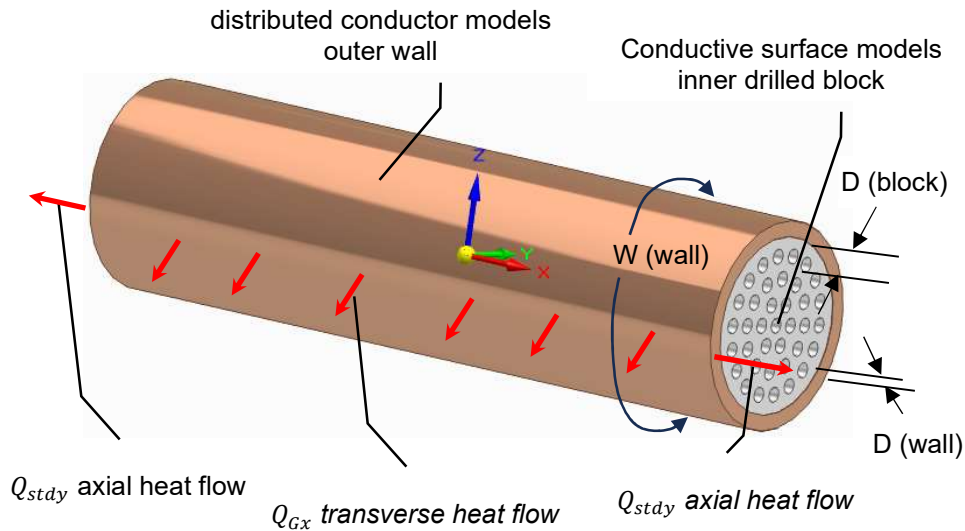
$$W = 2 * \text{Pi} * R_{\text{mean}}$$
$$D = \text{ThkOuterWall}$$

In terms of the user-defined output

$$R_{\text{mean}} = R_{\text{block}} + 0.5 * \text{ThkOuterWall} \quad 1.300\text{E-}02$$

$W$  and  $D$  correspond to the mean circumference and thickness of the outer wall, so the product  $W \times D$  is its cross-section area. In the Sage coordinate system (see Chapter Thermal Solids in the Sage User's guide)  $D$  is the  $y$ -thickness and  $W$  is the  $z$ -thickness. The meaning of that is that the heat conduction length is  $D$  for transverse heat flows ( $Q_{Gx}$  connections). There is no heat-flow connection for  $z$ -directed heat flow, which is reserved for connections to gas domains, not available in the distributed conductor component. So the sole purpose of  $W$  is to adjust the outer wall cross section area.

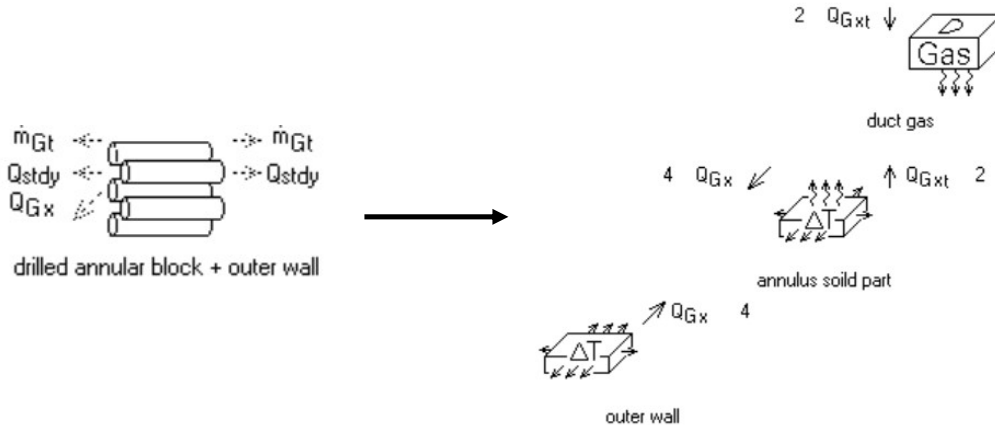
The physical heat exchanger looks something like this:



The  $Q_{stdy}$  connections can be used to transfer heat through the outer wall ends. The  $Q_{Gx}$  connection can be used to transfer heat through the outer wall cylindrical surface. One option is to attach the  $Q_{Gx}$  connector to a line heat source or line heater component with temperature or heat flux adjusted based on a separate analysis of external heat transfer.

### Drilled annular block bonded to conductive outer wall

This model extends the above Drilled Annular Block model by connecting it to an outer wall which serves as the ultimate connection to a heat source or sink. The appearance in Sage is similar to the above Drilled Annulus, except the key components are renamed and there is an additional distributed conductor component named Outer Wall:



The user-defined variables of the top-level component are

Inputs		
Rinner	annulus inner radius (m)	7.000E-03
Router	annulus outer radius (m)	1.400E-02



```

ThkOuterWall      outer wall thickness (m)                2.000E-03

Output
Am                annulus solid cross section area      3.299E-04
  Pi * (Sqr(Router) - Sqr(Rinner)) - Aflow

Recast
Twall = 0.5*(Sqrt(Sqr(Dtube) + 4*Am/(Ntube * Pi)) - Dtube)

```

Input D of the annulus solid part is recast as for the stand-alone Drilled Annular Block.

$$D = Router - Rinner$$

Inputs W and D of the outer wall are recast as

$$W = 2 * \text{Pi} * R_{\text{mean}}$$

$$D = \text{ThkOuterWall}$$

In terms of the user-defined output

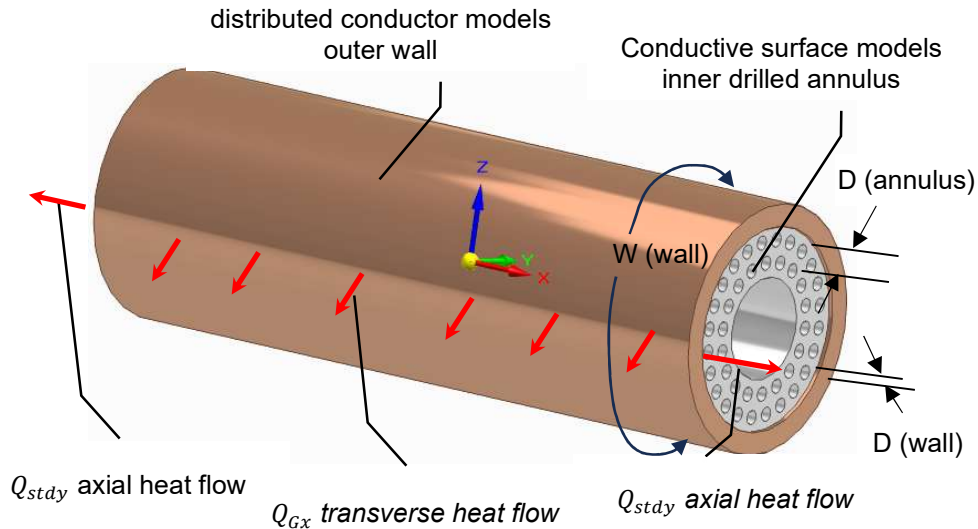
```

Rmean            mean radius                            1.500E-02
Router + 0.5*ThkOuterWall

```

W and D correspond to the mean circumference and thickness of the outer wall, so the product  $W \times D$  is its cross-section area. In the Sage coordinate system (see Chapter Thermal Solids in the Sage User's guide) D is the y-thickness and W is the z-thickness. The meaning of that is that the heat conduction length is D for transverse heat flows ( $Q_{Gx}$  connections). There is no heat-flow connection for z-directed heat flow, which is reserved for connections to gas domains, not available in the distributed conductor component. So the sole purpose of W is to adjust the outer wall cross section area.

The physical heat exchanger looks something like this:



The  $Q_{std}$  connections can be used to transfer heat through the outer wall ends. The  $Q_{Gx}$  connection can be used to transfer heat through the outer wall cylindrical surface. One option is to attach the  $Q_{Gx}$  connector to a line heat source or line heater component with temperature or heat flux adjusted based on a separate analysis of external heat transfer.

## Rectangular Channel Heat Exchangers

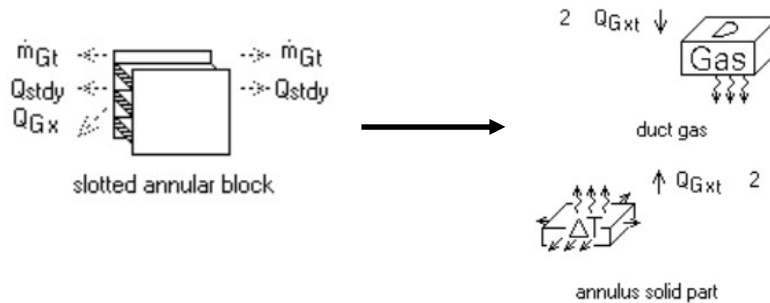
The rectangular-fins component can model heat exchangers comprising radial rectangular channels cut in a conductive material. It calculates a solid cross section area according to channel height, fin thickness and channel number inputs. You need to specify fin thickness (input  $T_{fin}$ ) to make the total solid cross section calculated by Sage (output  $A_{sec}$ ) equal the actual solid area  $A_m$ . This will be so when

$$t_f = \frac{A_m}{nh_i}$$

where  $t_f$  is fin thickness,  $n$  is the number of channels and  $h_i$  is the channel height. The examples below illustrate only the annular variations of this type of heat exchanger.

### Slotted annular block

In Sage, the model uses the *rectangular fins* and *distributed conductor* components, except they are renamed *slotted annular block* and *annulus solid part*:



The user-defined variables of the rectangular channel annulus are

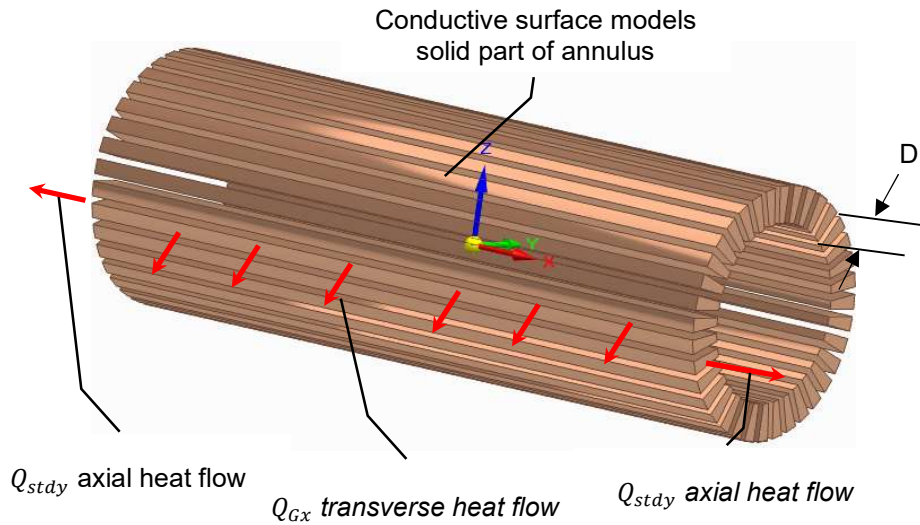
Inputs		
Rinner	annulus inner radius (m)	1.200E-02
Output		
Router	annulus outer radius	1.700E-02
Rinner + Hchan		
Am	solid cross section area	2.755E-04
Pi * (Sqr(Router) - Sqr(Rinner)) - Aflow		
Recast		
Tfin = Am / (Nchan * Hchan)		

In other words, the slots cut through the annular block from outer to inner radius. Input  $D$  of the annulus solid part is recast as

$$D = Hchan$$

Which is the radial fin conduction path length. This amounts to approximating the radial heat flow by linear heat flow through rectangular fins (see discussion in sample model HeatExchangers-SimpleIsothermal.scfn, Rectangular Fins section).

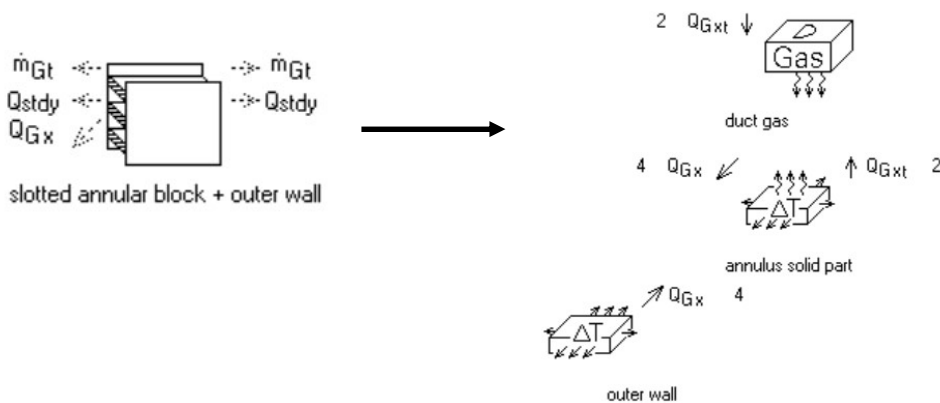
The physical heat exchanger looks something like this:



The  $Q_{stdy}$  connections can be used to transfer heat through the annulus ends. The  $Q_{Gx}$  connection can be used to transfer heat through the outer cylindrical surface (all fins simultaneously). One option is to attach the  $Q_{Gx}$  connector to a line heat source or line heater component with temperature or heat flux adjusted based on a separate analysis of external heat transfer.

### Slotted annular block bonded to conductive wall

This model extends the above Slotted Annular Block model by connecting it to an outer wall which serves as the ultimate connection to a heat source or sink. The appearance in Sage is similar to the above Slotted Annular Block except the key components are renamed and there is an additional distributed conductor component named Outer Wall:



The user-defined variables of the rectangular channel annulus are

Inputs		
Rinner	annulus inner radius (m)	1.200E-02
ThkOuterWall	outer wall thickness (m)	2.000E-03
Output		
Router	annulus outer radius	1.700E-02
Rinner + Hchan		
Am	solid cross section area	2.755E-04

$$\text{Pi} * (\text{Sqr}(\text{Router}) - \text{Sqr}(\text{Rinner})) - \text{Aflow}$$

Recast

$$\text{Tfin} = \text{Am} / (\text{Nchan} * \text{Hchan})$$

In other words, the slots cut through the annular block from outer to inner radius. Input D of the annulus solid part is recast as for the stand-alone Slotted Annular Block.

$$D = \text{Hchan}$$

Inputs W and D of the outer wall are recast as

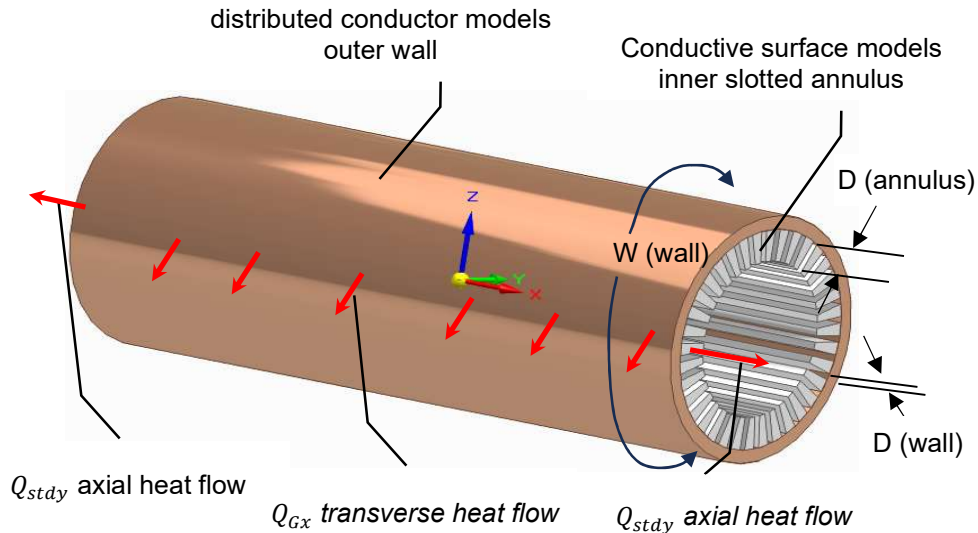
$$\begin{aligned} W &= 2 * \text{Pi} * \text{Rmean} \\ D &= \text{ThkOuterWall} \end{aligned}$$

In terms of the user-defined output

Rmean	mean radius	1.500E-02
Router + 0.5*ThkOuterWall		

W and D correspond to the mean circumference and thickness of the outer wall, so the product  $W \times D$  is its cross-section area. In the Sage coordinate system (see Chapter Thermal Solids in the Sage User's guide) D is the y-thickness and W is the z-thickness. The meaning of that is that the heat conduction length is D for transverse heat flows ( $Q_{Gx}$  connections). There is no heat-flow connection for z-directed heat flow, which is reserved for connections to gas domains, not available in the distributed conductor component. So the sole purpose of W is to adjust the outer wall cross section area.

The physical heat exchanger looks something like this:



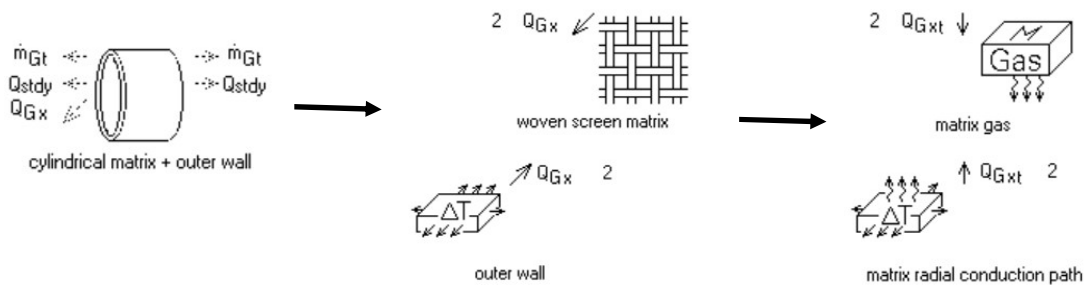
The  $Q_{stdy}$  connections can be used to transfer heat through the outer wall ends. The  $Q_{Gx}$  connection can be used to transfer heat through the outer wall cylindrical surface. One option is to attach the  $Q_{Gx}$  connector to a line heat source or line heater component with temperature or heat flux adjusted based on a separate analysis of external heat transfer.

## Porous Matrix Heat Exchangers

In the separate sample model HeatExchanger-ConductiveMatrix.scfn the conductive matrix was anchored to a line heat source. The conductive matrix (screens, random fibers or packed spheres) can also be anchored to the wall(s) of a conductive canister. The modeling is a bit different from the above heat-exchanger examples because the porous-matrix model components are not stand-alone components but rather reside only within canister components. The examples below illustrate cylindrical and annular variations of porous matrix heat exchangers.

### Cylindrical matrix bonded to conductive canister wall

In Sage, the model uses the *tubular canister*, *distributed conductor* and *conductive surface* components, except they are renamed *cylindrical matrix + outer wall*, *outer wall* and *matrix radial conduction path*:



The matrix-radial-conduction path  $Q_{Gx}$  connector has been bumped up one level for connection to the outer wall.

There are no user-defined variables in the top-level canister, woven-screen matrix or matrix gas.

Inputs D of the outer wall is recast to the parent canister wall thickness

$$D = W_{can}$$

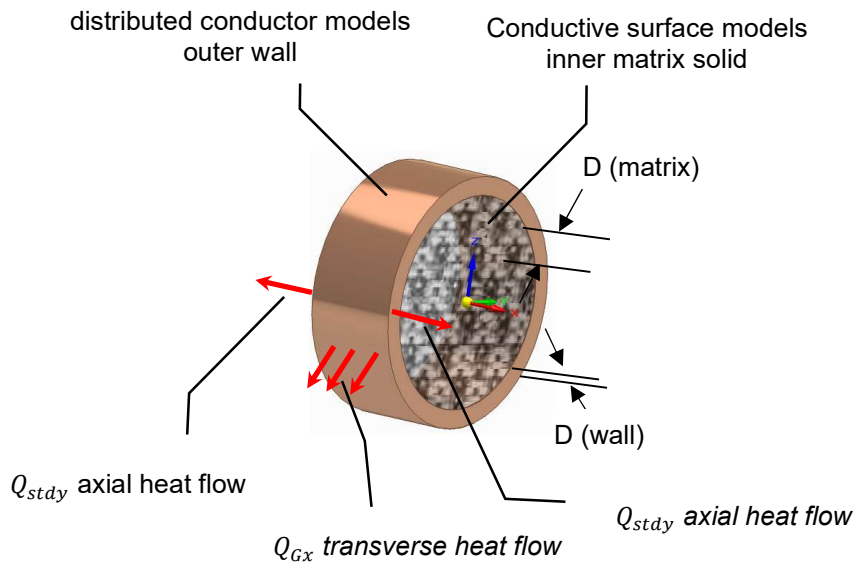
Sage calculates outer wall mean circumference  $\bar{w}$  automatically from parent canister inputs.

Inputs D of the matrix radial conduction path is recast to half the parent canister radius (1/4 the diameter)

$$D = 0.25 * D_{in}$$

See the documentation for sample model HeatExchanger-ConductiveMatrix.scfn for why that makes sense.

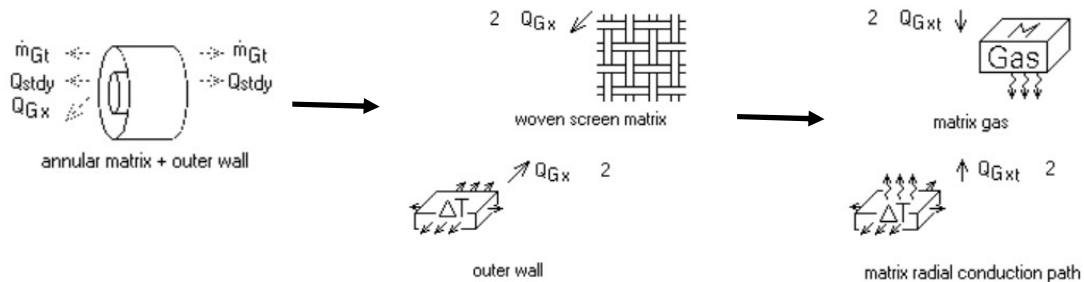
The physical heat exchanger looks something like this:



The  $Q_{stdy}$  connections can be used to transfer heat through the outer wall ends. The  $Q_{Gx}$  connection can be used to transfer heat through the outer wall cylindrical surface. One option is to attach the  $Q_{Gx}$  connector to a line heat source or line heater component with temperature or heat flux adjusted based on a separate analysis of external heat transfer.

### Annular matrix bonded to conductive canister wall

In Sage, the model uses the *annular canister*, *distributed conductor* and *conductive surface* components, except they are renamed *annular matrix + outer wall*, *outer wall* and *matrix radial conduction path*:



The matrix-radial-conduction path  $Q_{Gx}$  connector has been bumped up one level for connection to the outer wall.

There are no user-defined variables in the top-level canister, woven-screen matrix or matrix gas.

Inputs D of the outer wall is recast to the parent canister outer wall thickness

$$D = W_{out}$$

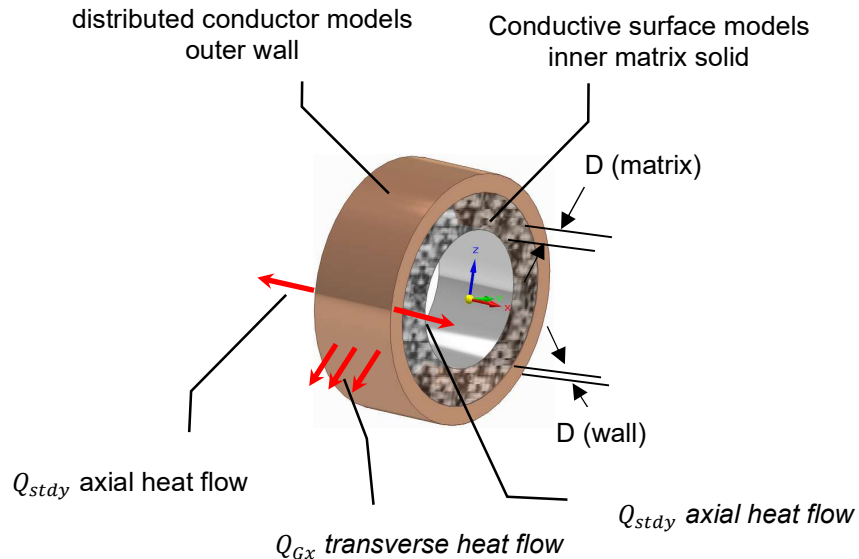
The inner wall thickness  $W_{in}$  is set close to zero, consistent with the assumption that only the outer wall serves as the conduction path to an external heat source or sink.

Inputs D of the matrix radial conduction path is recast as

$$D = 0.5 * ( (D_{out} - W_{out}) - (D_{in} + W_{in}) )$$

Which is the radial conduction path length. This amounts to approximating the radial heat flow by linear heat flow (see discussion in sample model HeatExchangers-SimpleIsothermal.scfn, Rectangular Fins section).

The physical heat exchanger looks something like this:



The  $Q_{stdy}$  connections can be used to transfer heat through the outer wall ends. The  $Q_{Gx}$  connection can be used to transfer heat through the outer wall cylindrical surface. One option is to attach the  $Q_{Gx}$  connector to a line heat source or line heater component with temperature or heat flux adjusted based on a separate analysis of external heat transfer.

**Annular Confusion** According to Sage's implementation of the annular canister a single distributed conductor represents both inner and outer walls. This is fine for modeling axial conduction. The confusion sets in for radial conduction because the inward direction for the inner wall and the outward direction for the outer wall correspond to the same direction in the distributed conductor. To eliminate this confusion the sample model sets the inner wall thickness  $w_{in}$  to zero (actually a very small number since it must be positive), allowing only the outer wall to participate in heat transfer. If you must model an inner wall, you could add a distributed conductor representing that wall in a nested canister inside the primary canister and connect it to the positive y-boundary of the matrix with a  $Q_{Gx}$  heat-flow connector.