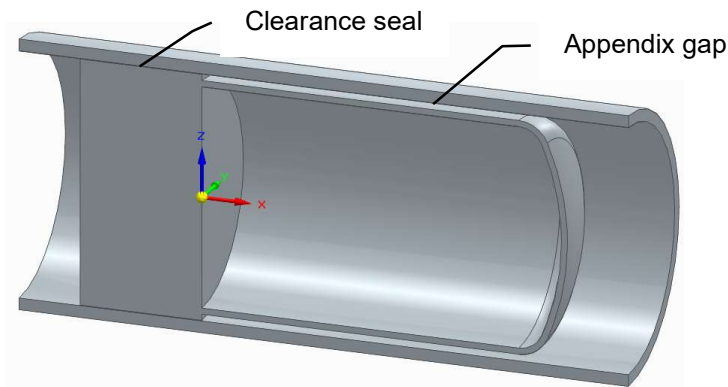


Sage Model Notes

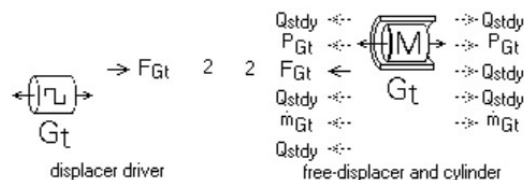
HollowDisplacer-Cylinder.scfn

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A model of a hollow-shell displacer moving within a cylinder.



This model is similar to the Piston-ClearanceSeal.scfn sample model, except that a displacer generally has a significant temperature gradient along its length so that shuttle heat transfer, solid conduction down the cylinder and displacer walls, and radiant heat transfer within the displacer shell are important. The gap between the cylinder wall and displacer shell typically consists of two parts, a clearance seal at the ambient temperature end and a larger-gap appendix at the hot or cold end. The basis for this model is the *free-piston and cylinder* component on the *Composite* page of the root-level component palette, renamed *free-displacer and cylinder* below:



To make this model fully functional it must be copied into another model containing two *generic cylinder* (AKA variable volume space) components (representing stirling-cycle compression and expansion spaces) with mating P_{Gt} connectors to connect to those emerging from the *free-displacer and cylinder*. Also required are some available mating m_{Gt} gas flow connectors and Q_{stdy} heat-flow connectors from heat exchangers and point temperature sources or sinks. The model is configured so that all negative directed Q_{stdy} connectors (left side) provide ambient temperature boundary conditions and all positive directed connectors (right) side provide cold-end (in case of a cooler) or hot-end (in case of an engine) boundary conditions.

The displacer driver is just a renamed *constrained piston* from the Gt Moving Parts palette. Its purpose is to drive the *reciprocator* within the *free-displacer and cylinder* via the F_{Gt} connector between the two. In effect, the *displacer driver* produces whatever force is required to make the *reciprocator* move with identical motion.

Note: when part of a free-displacer model it is possible to drive the reciprocator with springs and pneumatic forces by optimizing appropriate model variables. To verify you have done this correctly you can disconnect the displacer-driver force and make sure the model still runs the same, or close.

The *displacer driver* contains two user-defined inputs:

Xamp	displacer amplitude (m)	5.000E-03
Xarg	displacer phase angle (rad)	1.570E+00

And recasts the displacer motion as sinusoidal with that amplitude and phase:

```
FX = 0.000E+00...
(Xamp) Amp
(Xarg) Arg
```

The phase angle is by convention relative to the zero-phase reference of the model, which is often the piston phase angle, but can be some other reference signal assigned zero phase, such as drive current.

The *free-displacer and cylinder* contains user-defined inputs:

Mdis	reciprocating mass (kg)	5.000E-02
DdisCyl	displacer cylinder diameter (m)	2.500E-02
Drod	drive rod diameter (m)	5.000E-03
GapSeal	radial clearance gap (m)	2.500E-05
GapAppendix	radial clearance gap (m)	5.000E-04

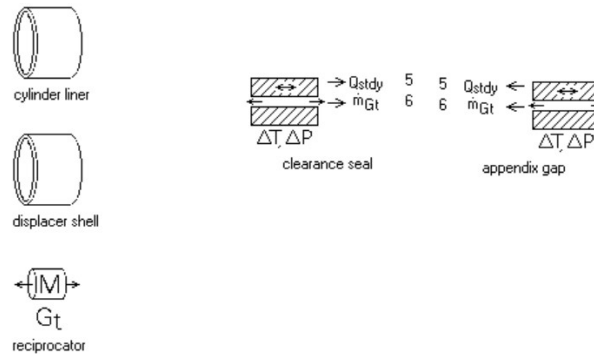
It recasts built-in input Dshell as

```
Dshell = DdisCyl
```

This makes more sense in a larger model where DdisCyl would likely be defined at a higher level. It also calculates two user-define outputs, useful below:

AdisCyl	displacer cylinder frontal area	4.909E-04
0.25*Pi * Sqr(DdisCyl)		
Arod	drive rod frontal area	1.963E-05
0.25*Pi * Sqr(Drod)		

Components inside the *free-displacer and cylinder* are



The *cylinder liner* defines a canister material and wall thickness via inputs

Solid	canister material	SS304
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Wcan wall thickness (m) 1.000E-03

but does not actually model heat transfer down the wall. That is done in the *appendix gap* as explained below.

This wall may or may not represent a pressure wall, depending on the physical hardware.

The **displacer shell** component defines a canister material and wall thickness via inputs

Solid canister material SS304
Wcan wall thickness (m) 1.000E-03

As for the cylinder liner, the *appendix gap* component takes care of thermal conduction down the shell. But the *displacer shell* does model rudimentary radiation heat transfer using the *radiation transport path* components.



The **radiation transport path** contains an input Emmis for the effective thermal emissivity of the two ends. See the Sage User's guide for more information (Help | PDF Manual). The present model contains a formulation presuming closely spaced radiation shields, a common configuration in displacers of high-temperature stirling engines, implemented in terms of the following user-defined inputs and recast:

Inputs
Emmis0 surface emissivity (NonDim) 5.000E-01
Nradshields number radiation shields (NonDim) 6.000E+00

Recasts

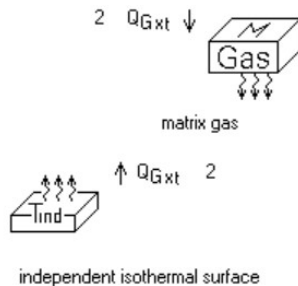
$$\text{Emmis} = 2 / ((\text{Nradshields} + 1) * (2 / \text{Emmis0} - 1) + 1)$$

The **clearance seal** is a renamed version of the *annulus shuttle/seal/appendix* component from the *Inter-Gap* page of the component palette. Child components within this component model the actual gas leakage. The component itself establishes the radial gap between shell and liner. It recasts the clearance-gap input to

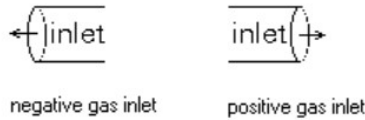
$$\text{Gap} = \text{GapSeal}$$

This makes more sense in a larger model where GapSeal would likely be defined at a higher level. The *clearance seal* also models shuttle heat transfer. More on that below.

Inside the *clearance seal* are, a *matrix gas* and an *independent isothermal surface* from the *Gas Domain* and *Cylinder Walls* pages of the component palette.



Within the **matrix gas** are negative and positive gas inlets, whose connectors have been bumped up to the *free-displacer and cylinder level* for connection to other components.



The **appendix gap** is a similar renamed version of the *annulus shuttle/seal/appendix* component, generally representing a larger clearance gap to minimize shuttle heat transfer loss and for mechanical tolerance reasons. The component recasts the radial clearance gap as:

$$\text{Gap} = \text{GapAppendix}$$

Where *GapAppendix* is generally much larger than *GapSeal*.

It models shuttle heat transfer between the *liner* and *shell*, which is implemented by built *Q_{stdy}* heat-flow connector arrows. The negative-facing connector is attached to the positive facing connector of the *clearance seal*. The positive facing connector and the negative facing connector for the *clearance seal* are bumped up to the free-displacer and cylinder level to be connected later. In other words the shuttle heat transfer for the *appendix gap* and *clearance seal* are connected in series. This works because Sage models shuttle heat transfer as an effective thermal conductance with the amount of heat transferred depending on the temperatures connected to the *Q_{stdy}* connectors. See the Sage manual for the detailed shuttle heat transfer formula (Help | PDF Manual).

Note: The lengths of the clearance seal and appendix gap are relative to the *Length* input of the parent *free-displacer and cylinder* component, as determined by inputs *XNeg* and *XPos*. In this model the negative and positive end boundaries are

clearance seal

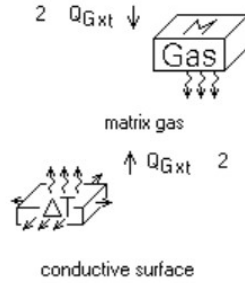
<i>XNeg</i>	parent-relative neg bnd [0, 1]	0.000E+00
<i>XPos</i>	parent-relative pos bnd [0, 1]	2.000E-01

Appendix gap

<i>XNeg</i>	parent-relative neg bnd [0, 1]	2.000E-01
<i>XPos</i>	parent-relative pos bnd [0, 1]	1.000E+00

Meaning that the *clearance seal* starts at the negative boundary with 20% of the parent length, followed by the appendix gap with 80% of the parent length ending at the positive boundary.

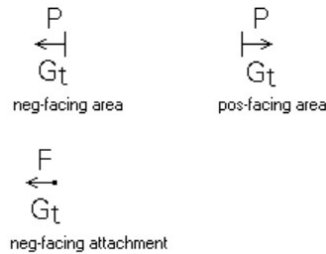
Inside the *appendix gap* are, a *matrix gas* and a *conductive surface* from the *Gas Domain* and *Cylinder Walls* pages of the component palette.



Inside the *conductive surface* are two Q_{stdy} heat flow connectors (one positive directed, the other negative) bumped up to the *free-displacer and cylinder* level. The *conductive surface* inherits the wall thicknesses and solid materials from the *cylinder liner* and *displacer shell* components so as to model the combined thermal conduction of both walls in parallel. No further inputs needed. The conduction so modeled corresponds to that portion of the length of the liner and shell specified by the X_{Neg} and X_{Pos} input variables.

Note: When modeling combined wall thermal conduction within the appendix gap do not also use *distributed conductors* within the *cylinder liner* or *displacer shell* to do the same thing. Doing so will model the same conduction paths twice.

Within the *reciprocator* of the *free-displacer and cylinder* are positive and negative facing area attachments.



Both have area inputs recast according to the convention that the variable-volume space connected to the pos-facing area sees the full displacer area, while the space connected to the neg-facing area sees the differential area between the displacer and drive rod.

Pos-facing area
 $A = A_{disCyl}$

Neg-facing area
 $A = A_{disCyl} - A_{rod}$

Their P_{Gt} connector arrows have been bumped up to the *free-piston and cylinder* level for connection there to variable-volume gas domains, such as the expansion and compression spaces in a stirling-cycle model.

Note: if the far end of the displacer drive rod also interacts with a variable-volume space of the model you can drop in another neg-facing area with input A recast to A_{rod} .

There is also a force connection, whose F_{Gt} connector arrow has also been bumped up to the *free-displacer and cylinder* level for connection to the *displacer driver*.