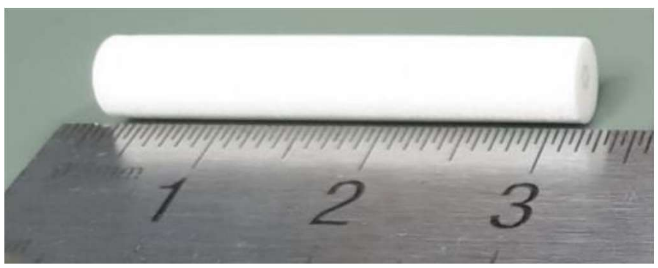
Sage Model Notes

Regenerator-GenericParallelFiber.scfn

D. Gedeon
11 December 2024

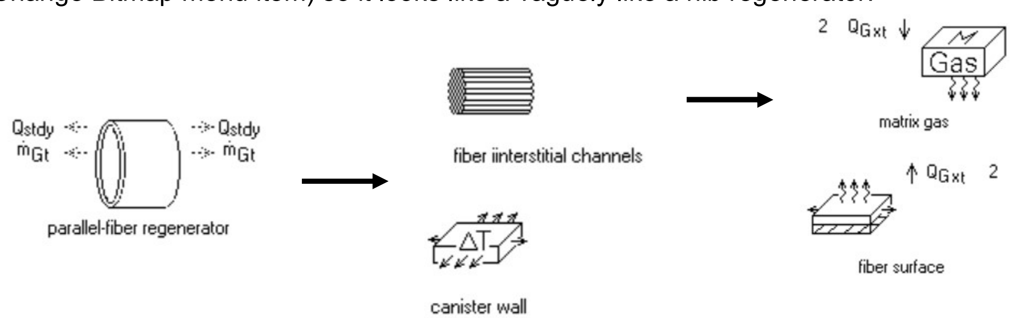
Sage has several built-in regenerator matrix options (*Matrices* page of component palette inside canisters) but the list is not comprehensive. To permit modeling non-standard matrix geometries Sage provides the *Generic Matrix* option, which allows you to specify your own coefficients and exponents within generic formulations for friction factor, Nusselt number, axial conduction enhancement and tortuosity. See chapter Heat Exchangers in the Sage User's Guide (Help | PDF Manual).

This model illustrates this process with a generic regenerator model for a parallel-fiber regenerator matrix, of the type Alex Veprik of CryoTech LTD has successfully incorporated into a small cryocooler[1]. This style of matrix is commercially available in the form of round cartridges containing bundles of nylon fibers, originally intended for nibs in felt tip marking pens.



An important consideration for parallel-fiber regenerators is that the fiber material must have low thermal conductivity to avoid short-circuiting regenerator performance. There is a continuous solid conduction path along the fibers in the flow direction (direction of thermal gradient). Plastic fibers are ideal. Stainless steel or other metallic fibers will not perform as well as they do in cross-flow geometries (e.g. stacked screens).

The Sage model consists of a cylindrical canister, renamed *parallel-fiber regenerator*, containing a generic matrix component renamed *fiber interstitial channels* and for a visual clue that it is not an ordinary generic matrix has a revised bitmap representation (Edit -> Change Bitmap menu item) so it looks like a vaguely like a nib regenerator:



[1] *Lowcost cryogenic technologies for high-operating temperature infrared imaging*, A. Veprik, D. Gedeon, R. Radebaugh, R. Refaeli, S. Kurucz, L. Bunin, Proc. SPIE 12534, Infrared Technology and Applications XLIX, 125340C (13 June 2023); doi: 10.1117/12.2664389

The *canister wall* is a renamed *distributed conductor* component that models canister wall conduction. The *fiber surface* is a renamed *rigorous surface* component that models the solid part of the fibers in intimate contact with the matrix gas. The $\dot{m}_{Gt}$ connectors at the canister level are bumped up from the inlets within the *matrix gas* component and the Q_{stdy} heat-flow connectors come from the heat-flow ends within the distributed conductor.

To make this model fully functional it must be copied into another model containing two mating $\dot{m}_{Gt}$ gas flow connectors from heat-rejecting and heat-accepting heat exchangers and two mating Q_{stdy} heat-flow connectors from point temperature sources or sinks at the temperatures of the regenerator endpoints.

The *fiber interstitial channels* (generic matrix) component defines porosity β as a built-in input and inherits canister area A_c from the parent canister component. It contains a user-defined input for the fiber diameter $\mathrm{d_w}$. The hydraulic diameter input d_h is then recast as follows: (see below for math)

```
Inputs
  Dw                fiber diameter (m)                        2.500E-05
Recasts
  Dhyd = Porosity*Dw/Alpha
Outputs
  Alpha             solid/total volume                        2.100E-01
    1-Porosity
```

There are no other user-defined variables in this model. Rather, a number of build-in coefficient and exponent inputs of the *matrix gas* and *interstitial channels* components are set according to the flow geometry.

Setting Generic Matrix inputs

The presumption of this model is that parallel fibers are hydrodynamically similar to flow through equilateral-triangular ducts, which is roughly the shape of ideal passages between perfectly straight fibers packed together in a perfect hexagonal pattern (each fiber tangent to its 6 neighbors). This is not a very good assumption based on the actual matrix structure (see below) but it serves as a starting point, pending availability of experimental test data for the actual geometry.

In other words, since the flow resistance and heat-transfer correlations of this model are not backed by actual test data (as are Sage's built-in matrix options) you should expect to spend some time calibrating the model inputs (below) before hardware performs as expected.

Assuming the Reynolds number is low and the gas thermal penetration depth large compared to the hydraulic diameter, the flow can be considered laminar with negligible entry length for developing flow. Kays and London[2] tabulate analytic solutions for simple geometries including an equilateral triangle duct. Below are the Kays and London values for laminar developed flow in equilateral triangle ducts, compared to circular tubes:

[2] Kays and London, Compact Heat Exchangers, 3rd edition, (1984), fig 6-1, p.120

	Nusselt number (constant heat flux boundary)	Friction factor (x 4 to convert to hydraulic diameter form)
Circular tubes	4.36	64
Triangular duct	3.11	53.3

In the Sage model the triangular-duct values are implement as follows. See the Generic Matrix section of the Sage User's Guide for more details (Help | PDF Manual).

Friction factor

Matrix gas inputs:

```
FdC1          C1 of f = C1 + C2 Re^m + C3/Re           0.000E+00
FdC2          C2 of f = C1 + C2 Re^m + C3/Re           0.000E+00
FdC3          C3 of f = C1 + C2 Re^m + C3/Re           5.300E+01
FdM           m of f = C1 + C2 Re^m + C3/Re            0.000E+00
```

Corresponding to

$$f = 53/Re$$

Nusselt number

Matrix gas inputs:

```
NuC1          C1 of Nu = C1 + C2 Re^m Pr^n             3.100E+00
NuC2          C2 of Nu = C1 + C2 Re^m Pr^n             0.000E+00
NuM           m of Nu = C1 + C2 Re^m Pr^n              0.000E+00
NuN           n of Nu = C1 + C2 Re^m Pr^n              0.000E+00
```

Corresponding to

$$Nu = 3.1$$

Gas axial conduction enhancement ratio

Matrix gas inputs:

```
KrC1          C1 of Nk = C1 + C2 Re^m Pr^n             1.000E+00
KrC2          C2 of Nk = C1 + C2 Re^m Pr^n             0.000E+00
KrM           m of Nk = C1 + C2 Re^m Pr^n              0.000E+00
KrN           n of Nk = C1 + C2 Re^m Pr^n              0.000E+00
```

Corresponding to

$$N_k = 1$$

Matrix tortuosity factor

Generic matrix inputs *(fiber interstitial channels)*:

```
FsC1          C1 of fs = C1 + C2 (ks/kg)^m fs_Maxwell  1.000E+00
FsC2          C2 of fs = C1 + C2 (ks/kg)^m fs_Maxwell  0.000E+00
FsM           m of fs = C1 + C2 (ks/kg)^m fs_Maxwell   0.000E+00
```

Corresponding to

$$f_s = 1$$

The formulas used to calculate the hydraulic diameter from the fiber diameter d_w, canister area A_c and porosity β are below:

fill factor (solid volume / total volume)

$$\alpha = 1 - \beta$$

number fibers in cross section

$$N_w = \frac{4\alpha A_c}{\pi d_w^2}$$

wetted perimeter

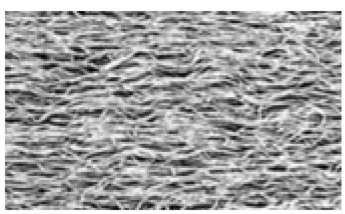
$$s_x = N_w \pi d_w = \frac{4\alpha A_c}{d_w}$$

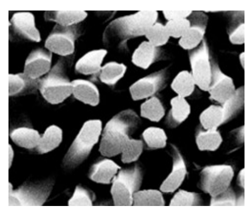
hydraulic diameter (4 x void area / wetted perimeter)

$$d_h = \frac{4\beta A_c}{s_x} = \frac{\beta d_w}{\alpha}$$

Fiber Packing Reality

Based on these micrographs the actual matrix geometry differs significantly from hexagonal close-packed straight fibers:





The fibers are curly and not perfectly packed together. Instead, there are random gaps between fibers that produce a network of non-uniform inter-connected flow channels. These will produce microscopic flow fluctuations with effective enhancements of transport properties similar to stacked-mesh regenerators but not as extreme because the local flow-path expansions and contractions between mostly parallel fibers are much less than the wires in cross flow of a stacked mesh.

Experience shows that some degree of transverse mixing of the regenerator flow (in planes perpendicular to the flow direction) is beneficial as it can suppress macroscopic flow circulations without unduly increasing flow resistance or longitudinal thermal diffusivity.

On a smaller scale, the effect of such irregularities is similar to the eddies in turbulent flow which tend to transfer momentum and heat amongst themselves, leading to increased flow friction and transverse heat transfer between the gas and matrix (good) and also increased thermal diffusivity in the flow direction (bad). These effects are not in the above model but could be added by modifying friction factor, Nusselt number or axial-conduction enhancement inputs to fit available data.