Sage Model Notes

4KThreeStage-RegenSizer.scfn

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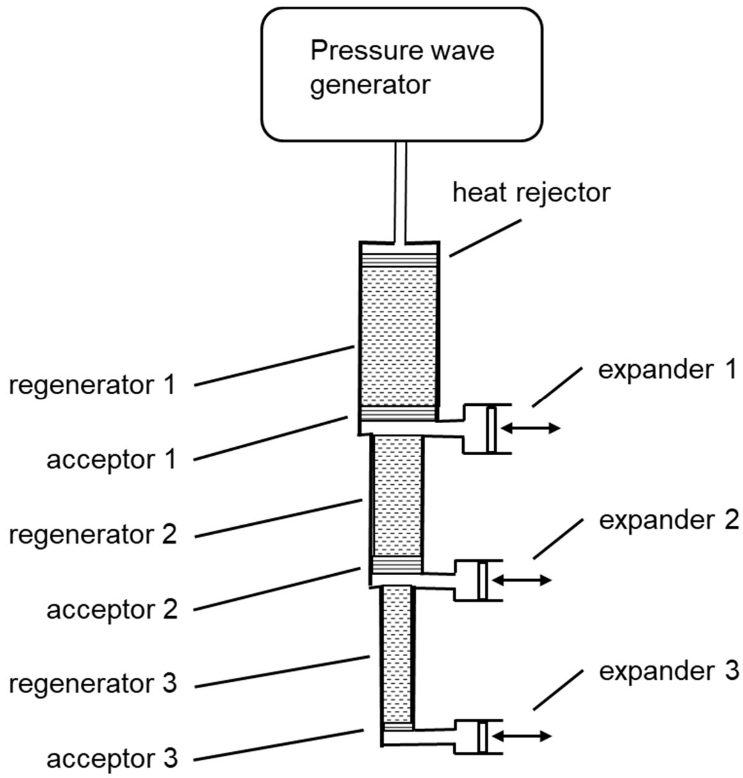
A simplified three-stage 4 K cryocooler with idealized expander volumes in each stage, designed to allow relatively fast regenerator sizing without worrying about the detailed components providing the expander functionality. In other words, the model contains no displacers, buffer tubes, bypass valves, orifi, intertance tubes, reservoirs, etc. The model schematic looks like this:



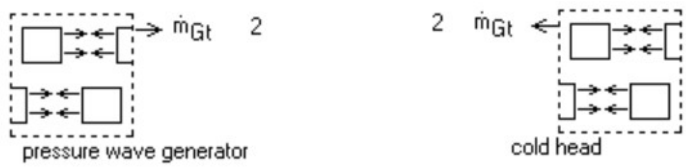
This model is essentially a stripped-down version of the **HiFreqPTR-ThreeStage.ltc** model.

Divide and Conquer A 4 K, three-stage model like HiFreqPTR-ThreeStage.ltc with all the details can be difficult to optimize. Unless you start out with a state that is pretty close to a viable cryocooler, convergence may be problematic or the optimizer may wander around hopelessly lost. This model is not so sensitive. It solves faster, converges more reliably and is much simpler.

The expander components are idealized piston / cylinder submodels without any thermal conduction losses. Basically you or the Sage optimizer can adjust the swept volume amplitude and phase angle of each one separately to maximize the cooling powers in the acceptor heat exchangers. The expanders extract PV power from the thermodynamic cycle similar to the way a displacer-type expander extracts PV power in a stirling-cycle cyocooler, except without any of the mechanical details. The net result is a model that allows you to focus on regenerator design. After that you can replace the expanders as needed with pulse-tubes, pressure phase shifting components or whatever, according to the hardware design you are working on. More on that later.

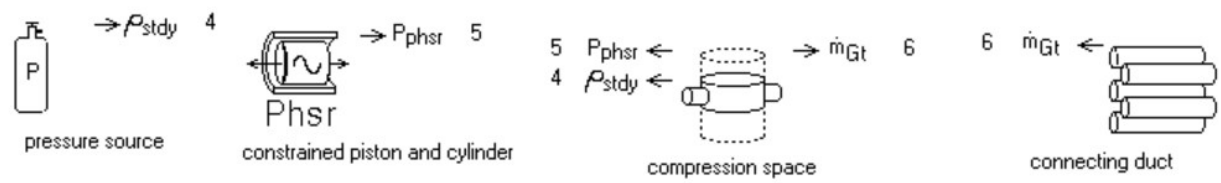
Root Model

The root model defines the operating frequency and is organized into two submodels:



Pressure Wave Generator

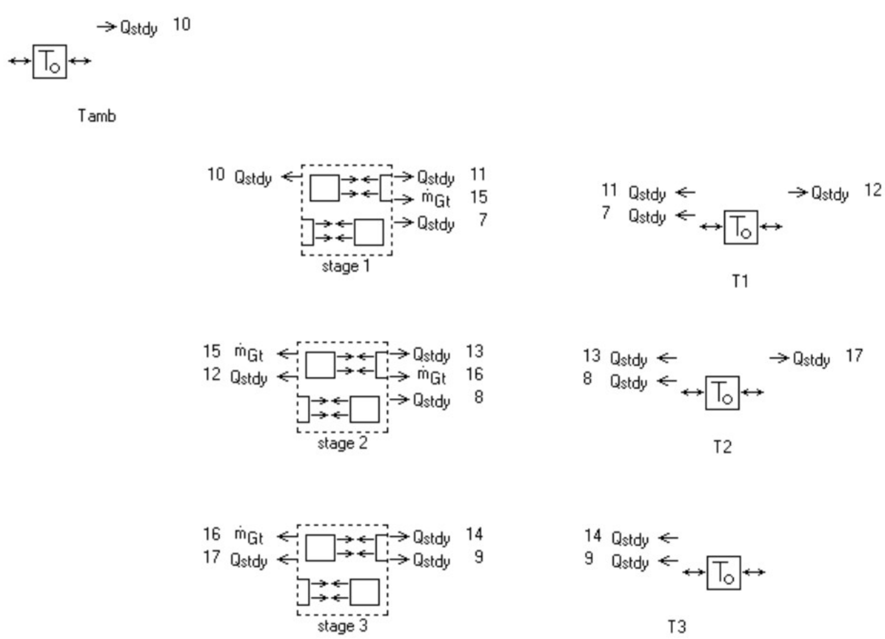
A *constrained piston and cylinder* provide a sinusoidal volume displacement to a *compression space* connected to the cold head by a *connecting duct*:



The *pressure-source* defines the time-average pressure (2 MPa).

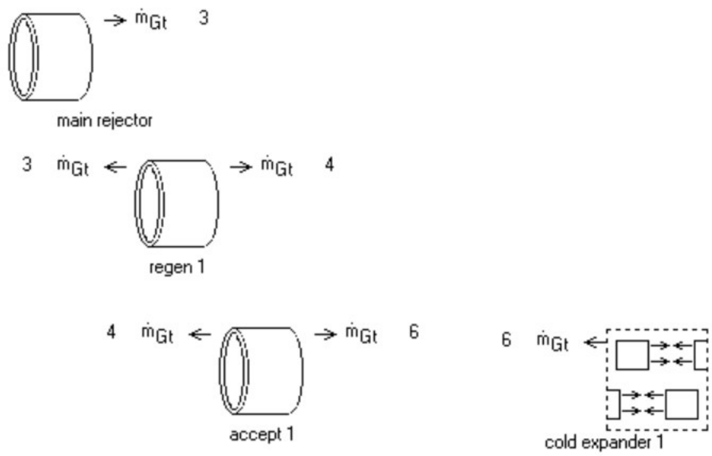
Cold Head

The cold head is broken into *stage 1, stage 2* and *stage 3* submodels with a number of temperature sources for anchoring the components within:



The ambient temperature (*Tamb*) anchors the main *heat rejector*. *Stage 1* temperature (*T1*) anchors cold heat exchanger in *Stage 1*, and so forth.

The components in the *stage 1*, *stage 2* and *stage 3* submodels are similar, corresponding to the above schematic. For example, these are the *stage 1* components:



Discharge flow from the *pressure wave generator* enters at the negative (left) boundary of the main warm heat exchanger (*main rejector)*, then proceeds through the regenerator (*regen 1*), cold heat exchanger (*accept 1)* and then splits into two streams, one proceeding on to stage 2 and the other terminating in the expander (*cold expander 1)*. During the suction part of the cycle the flows are reversed. The expander is similar to the expansion space of a stirling cooling cycle, phase-shifting the flow relative to the pressure and extracting PV power, the gross cooling power of the refrigeration cycle according to a first-law energy balance. The components are arranged in rows, with colder components at the bottom. The “1” suffix in all the component names designates that this is the first stage.

Each stage has a user-defined input:

Dreg	regenerator ID (m)	5.729E-02

The diameters of all canister components within that stage are recast in terms of Dreg.

The root-level model contains the following summary outputs for the PV power supplied by the *pressure wave generator* and the net cooling powers at the various temperature sources:

```
Wpv            pv power input                      5.000E+02
  Wpis
Qlift1         first stage heat lift               2.000E+01
  QT1
Qlift2         second stage heat lift              4.000E+00
  QT2
Qlift3         third stage heat lift               2.947E-01
  QT3
```

Expander Submodels

Each expander submodels contain a generic cylinder (variable-volume space) with an adiabatic wall and a constrained piston to provide a prescribed volumetric displacement.

There are some inputs associated with the mass flow rate boundary condition at the expander inlet and some outputs that translate those inputs into swept volume amplitude for internal use. The following are for the stage 3 expander:

Inputs
```
  MdotAmp          mass flow rate amplitude (kg/s)          8.479E-03
  MdotPhase        mass flow rate phase (deg)               7.361E+01
  Rhom             time-average density (kg/m3)             1.650E+02
  Dexpander        effective piston diameter (m)            1.000E-02
  Angle90           (deg)                                   9.000E+01
```
Outputs
```
  VdotAmp          volumetric flow amplitude                5.139E-05
    MdotAmp / Rhom
  Vamp             volume amplitude                         2.726E-07
    VdotAmp / (2*Pi*Freq)
  Aexpander        effective expander area                  7.854E-05
    0.25*Pi * Sqr(Dexpander)
```

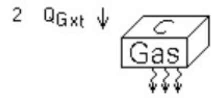
The reason that MdotAmp and MdotPhase are inputs is because they are fundamental solution variables, directly comparable to the FRhoUA… outputs available for all gas domains. That makes it relatively easy to insert an equivalent expander component into cryocooler model at any location. Volumetric flow amplitudes and phases are only available in variable-volume spaces, so are not as convenient. Input Rhom is important for calculating the equivalent volumentric flow amplitude Vamp.

The generic cylinder recasts inputs so its volume is always 50% larger than the swept volume amplitude.
Recasts
```
  Volume = 1.5*Vamp
  Swet = Aexpander
```

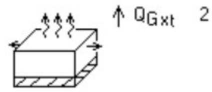
The constrained piston recasts amplitude, phase and frontal area (negative-facing area) according to the above inputs

```
  Xphase = MdotPhase - Angle90
  Xamp = VAmp /  Aexpander
  A = Aexpander
```

The gas inside the generic cylinder exchanges heat with an adiabatic thick-surface (not connected to any external temperature anchor).



cylinder-space gas



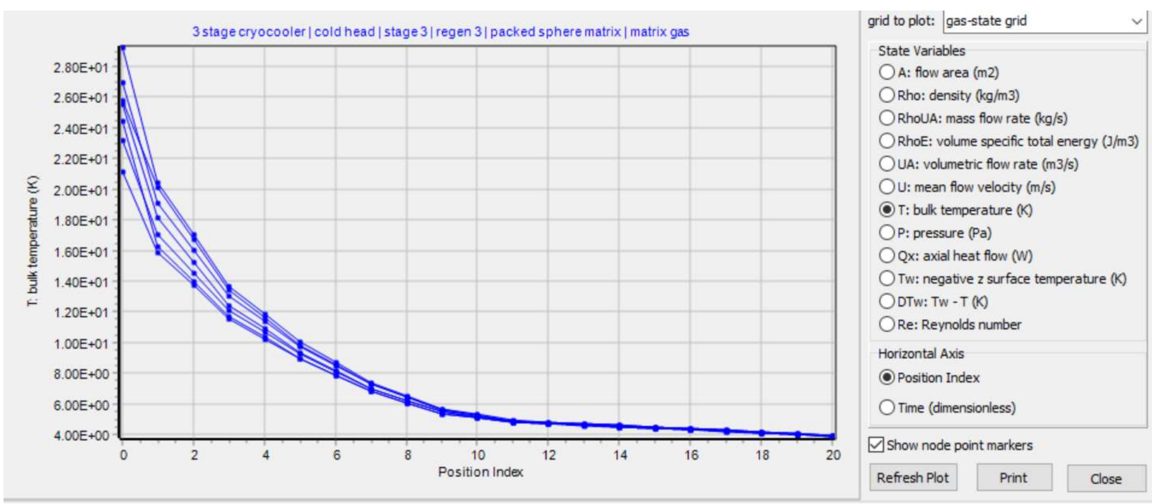
thick surface

That means the solved temperature will float according to the temperature of the upstream heat exchanger. But for best model convergence the initial temperature should be set close to that heat exchanger temperature using the generic cylinder input

```
 Tinit            initial temperature (NonDim, K)          unit spline...
    (0.000E+00, 4.000E+00)
    (1.000E+00, 4.000E+00)
```

4 K Regenerator

The 4 K regenerator requires special mention because of the low heat capacity of regenerator materials, which can cause unusually large temperature variations with time (especially at low frequency), and the pressure-dependent component of helium enthalpy, which tends to produce a concave temperature distribution. Both are visible in this plot of the stage 3 regenerator temperature distribution.



There is a detailed discussion of these effects in the model notes for the HiFreqPTR-ThreeStage.ltc model. To deal with these effects, the present model employs 10 spatial nodes in the regenerator, compared to 5 nodes for the stage 1 and 2 regenerators, which have nearly linear temperature distributions. In a more detailed cryocooler model a useful strategy for dealing with the concave temperature distribution is to somehow introduce a small DC mass-flow-rate bias to the time-varying mass flow rate, directed toward the cold end. The result is a steady stream of helium cooled to 4 K, emerging from the end of the regenerator by a process reminiscent of Joule-Thompson cooling. What might one do with this 4K helium stream? At any rate, the present model does not introduce any DC flow to avoid complicating the model.

Optimization

There are quite a few optimized variables for this model. The objective is to maximize the 4 K cooling power (Qlift3) subject to 500 W PV power delivered by the constrained piston to compression space of the pressure wave generator (Wpv) and some arbitrary cooling power values for the other stages.

Optimized are:

- Piston diameter for the pressure wave generator (amplitude fixed at 5.0 mm) with compression space mean volume adjusted to maintain a 50% clearance volume at *top dead center*.
- Interstage temperatures, subject to not-to-exceed temperature constraints.
- Regenerator diameters and lengths.
- Regenerator porosity or particle diameter, depending on the type of regenerator.
- Expander mass flow rate amplitudes and phases.

Heat exchanger lengths are fixed at convenient values except for the main rejector length, which is optimized. Heat exchangers are all modeled as copper screens with heat flow to the outer perimeter.

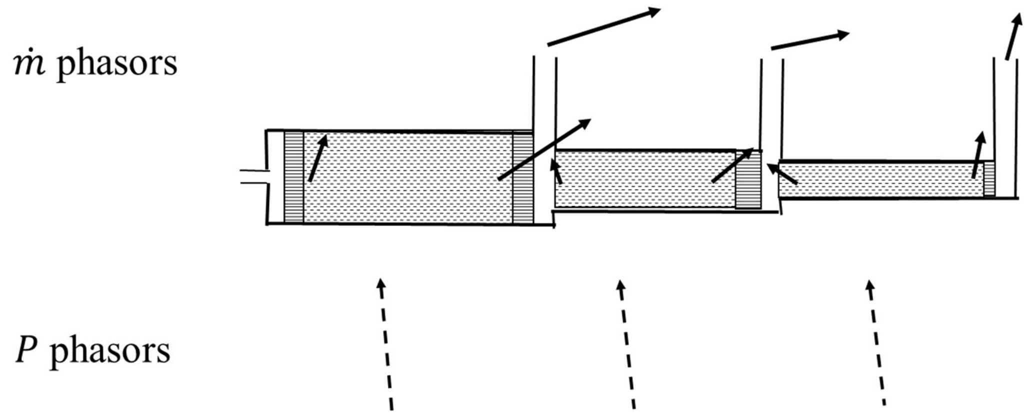
Carrying On From Here

One strategy for morphing this model into a more realistic representation of actual cryocooler hardware is to replace the idealized expander components one at a time with appropriate physical components, starting with expander 1. It may be a good idea to first model these physical expanders components in separate models, optimizing their design variables in an attempt to match the pressure and flow boundary conditions of the replaced idealized expander. Afterward you can copy the entire new expander into the larger model and re-optimize. In this way if something goes wrong you will have a good idea where the problem is and may be able to figure out how to correct it. You can copy and paste model components between models using the copy and paste tools of the Sage toolbar.

Note: Copy-and-pasting of model components in Sage does not use the Windows operating system clipboard, but as of Sage version 13 it works much the same. You can copy from one instance of Sage to another instance. Even after you have closed the first instance.

Mechanical expanders There should be no problem implementing this strategy for mechanical expanders that employ moving pistons or diaphragms driven by linear actuators. Possibly cold actuators or warm actuators separated from the cold parts by a thermal buffer tube or insulating piston shell. Such expanders can be tuned as a spring-mass-damper resonant system to any pressure-velocity phase relationship you want by balancing inertial, spring and actuator forces.

Pneumatic expanders It is more difficult with pneumatic expanders, consisting of pulse tubes (buffer tubes), warm heat exchangers, inertance tubes, bypass valves, orifi, reservoirs, etc., with no moving mechanical parts. To understand why, recall the general rule of thumb for a stirling thermodynamic cycle that the pressure and mass flow rate should be in phase somewhere in the regenerator for optimal performance (giving most PV power flow for least thermal loss). In this model, to optimize all three regenerators simultaneously the optimizer decided to divert mass flow rate into the expanders in such a way as to compensate for the tendency of mass flow rate phase to progressively lag from the first to the third stage regenerators as a result of their volume (compliance). This is clear in the diagram below which shows how the expander mass flow phasors shift the mass flow phasors at the entrance of the stage 2 and stage 3 regenerators to be more in-phase with the pressure phasor compared to the exit of the previous regenerator.



As a result, the mass-flow phasors of the stage 1 and stage 2 expanders lag the pressure phasors by more than the usual amount for a single-stage cooler — around 80 degrees. One problem is that the volume (compliance) of pneumatic components tend to do the opposite — make the mass flow rate phase lead the pressure phase. To compensate for that, one can add fluid inertia (inertance) with inertance tubes (long narrow tubes). But inertance tubes come with flow resistance which create a PV power flow in the circuit. It may be infeasible to add sufficient inertance without introducing excessive PV power flow (cooling power). Another problem is that it may be infeasible to achieve a high enough inlet impedance (high enough pressure amplitude relative to the required mass flow rate).

In the HiFreqPTR-ThreeStage.ltc model from which this model was derived the mass flow rate phasors lag the pressure phasors in the three stages by 38, 67, 20 degrees respectively, and the pressure amplitude is only about half as much.