Sage Model Notes

HiFreqPTR-ThreeStage.scfn

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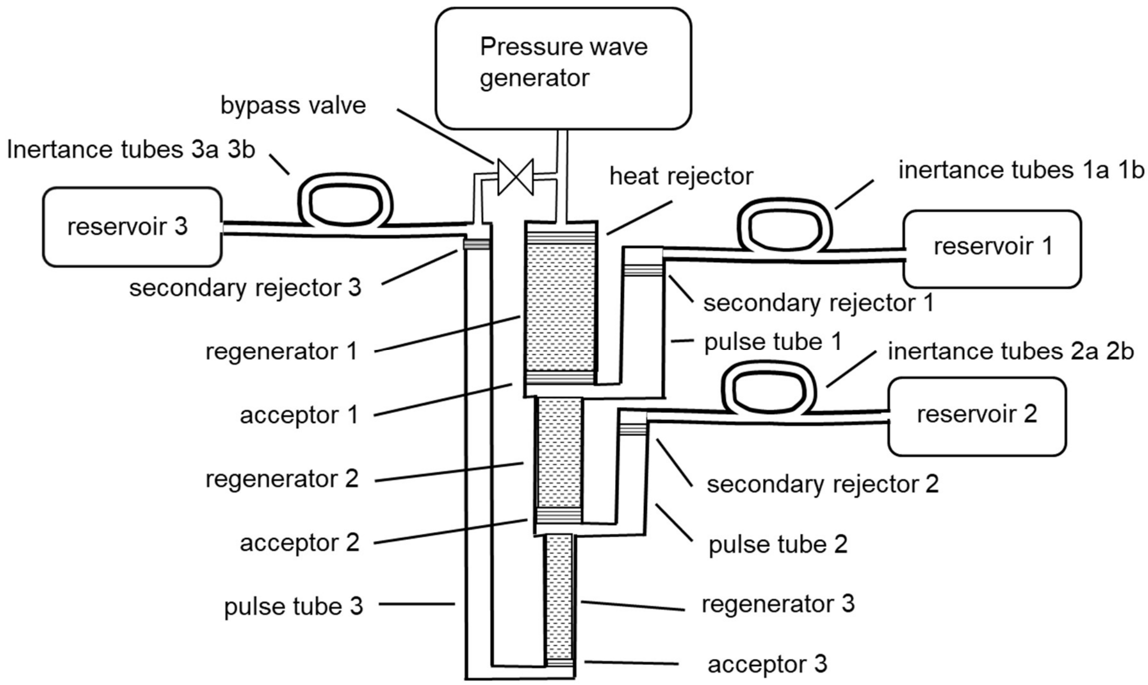
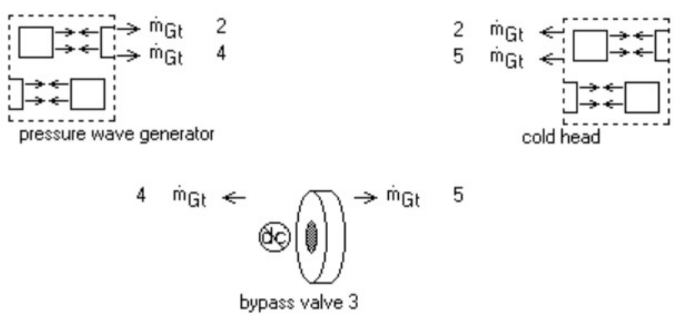
2 September 2021 (revised 16 August 2024)

A 4 K three-stage high-frequency (30 Hz) pulse-tube cryocooler. The model schematic looks like this:



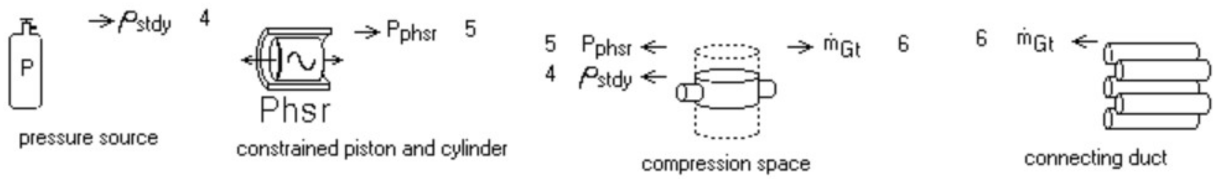
The root-level model is organized into submodels:



The root model includes a *bypass valve* (aka double-inlet valve) which connects the *pressure wave generator* output to the *stage 3* secondary rejector exit, as shown in the above schematic. The bypass valve helps to control the velocity amplitude within the cold head relative to the pressure amplitude — along the continuum between the velocity node of a standing wave and a traveling wave. With a properly tuned bypass valve the 4 K cooling power of this model is almost twice as much as the same model without the bypass valve. In Sage the *bypass valve* is implemented as a porous-plug orifice with the ability by numerical fiat to also control DC flow. The DC flow setpoint of the *bypass valve* is not zero in order to improve the 4K cooling power in the stage 3. Implementing an equivalent DC flow control in actual hardware may require a separate, smaller, valve or orifice in parallel with the main bypass valve, including a check valve to control the flow direction.

Pressure Wave Generator

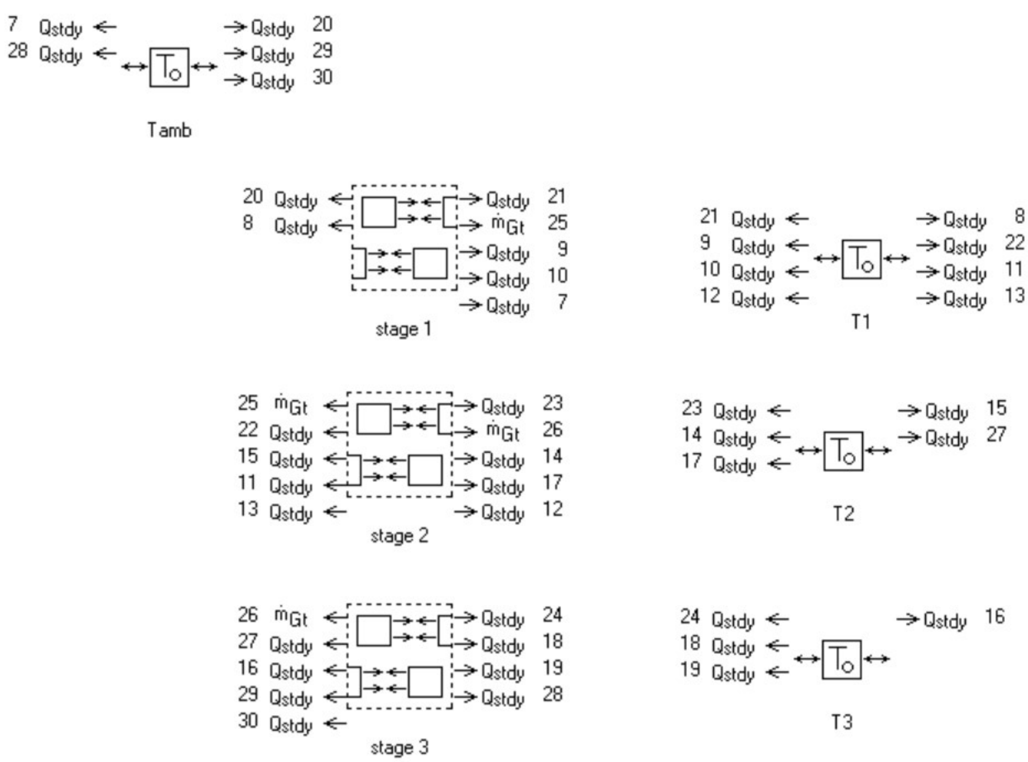
A *constrained piston and cylinder* provide a sinusoidal volume displacement to a *compression space* connected to the cold head by a *connecting duct*:



The *pressure-source* defines the time-average pressure (2 MPa).

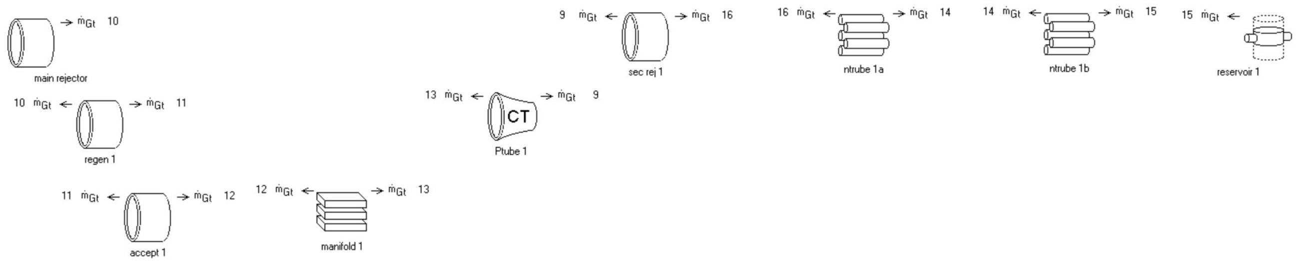
Cold Head

The cold head is broken into *stage 1, stage 2* and *stage 3* submodels with a number of temperature sources for anchoring the components within:



The ambient temperature (*Tamb*) anchors the main *heat rejector*, as well as the components at the warm-end of the *stage 1* pulse tube. *Stage 1* temperature (*T1*) anchors the warm end of the *stage 2* pulse tube. The s*tage 2* temperature (*T2*) could anchor the warm end of the *stage 3* pulse tube except that the connection to the bypass valve requires instead that ambient temperature (*Tamb*) anchor it, to prevent flow between different temperatures through the valve.

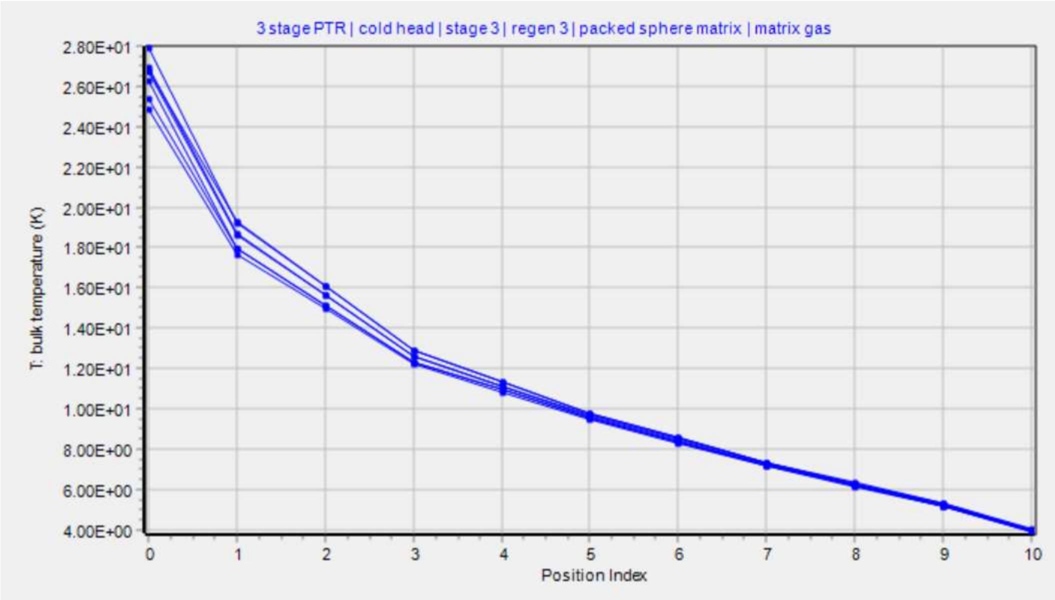
The components in the *stage 1*, *stage 2* and *stage 3* submodels are similar, corresponding to the above schematic. For example, these are the *stage 1* components:



Discharge flow from the *pressure wave generator* enters at the negative (left) boundary of the main warm heat exchanger (*main rejector)*, then proceeds through the regenerator (*regen 1*), cold heat exchanger (*accept 1)*, manifold (*manifold 1*), pulse tube (*ptube 1*), secondary warm heat exchanger (*sec rej 1*), inertance tubes *(ntube 1a, ntube 1b*) and into the reservoir (*reservoir 1)*. During the suction part of the cycle the flows are reversed. The inertance tubes serve to phase-shift the flow relative to the pressure so as to accomplish an approximation of the stirling cooling cycle — roughly speaking, so the pressure variation is in phase with the velocity variation in the regenerator. In *stage 3* there is a *bypass inlet* attached to the secondary rejector (*sec rej 3*) that serves to redirect the flow to or from the bypass valve (*bypass valve* 3) at the root level. The components are arranged in rows, with colder components at the bottom. The “1” suffix in all the component names designates that this is the first stage.

Third Stage in Particular

In *stage 3* the non-ideal-gas properties of helium are significant. One way they show up is in the concave-up regenerator temperature profile:



The reason has to do with the pressure dependent component of enthalpy flow, which is the second term in the formulation below for helium enthalpy transport

$$\dot{H} = C_p\langle\dot{m}(T - T_0)\rangle + C_T\langle\dot{m}(P - P_0)\rangle$$

$\dot{H}$ is enthalpy transport carried with a mass flow rate $\dot{m}$ past a point of observation with mean temperature and pressure T_0 and P_0. Symbol $\langle\rangle$ stands for time-average. C_p is the usual specific heat at constant pressure, familiar from ideal gas theory. C_T is the specific heat at constant temperature, defined in terms of the mass-specific enthalpy h by

$$C_T = \left(\frac{\partial h}{\partial P}\right)_T$$

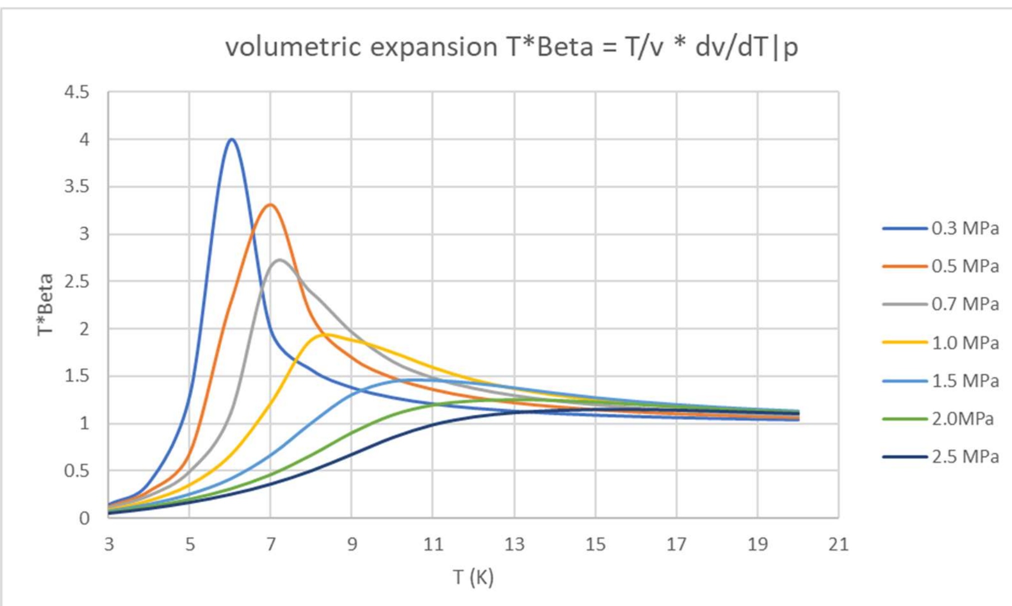
Thermodynamic textbooks derive an equivalent form for the partial derivative on the right as

$$\left(\frac{\partial h}{\partial P}\right)_T = v\left(1 - \frac{T}{v}\left(\frac{\partial v}{\partial T}\right)_P\right)$$

where v is specific volume $(1/\rho)$. The second factor on the right is usually written as $1 - T\,\beta$, where β is the coefficient of expansion

$$\beta = \frac{1}{v}\left(\frac{\partial v}{\partial T}\right)_P$$

The following plot shows $T\beta$ for helium 4, as a function of temperature, at various pressures.



T*Beta plot from NIST Refprop software

While a regenerator blocks the temperature-dependent part of enthalpy flow to the degree that it suppresses the temperature swing $T - T_0$, it always remains transparent to any pressure-dependent enthalpy flow, regardless of heat-transfer effectiveness. The temperature-dependent part of enthalpy flows toward the cold end of the regenerator ($T - T_0$ in phase with $\dot{m}$) and always constitutes a *loss* to net heat lift. The pressure-dependent part of enthalpy, on the other hand, can flow either direction, depending on whether $T\beta > 1$ or $T\beta < 1$. For the case $T\beta > 1$ (giving $C_T < 0$), the pressure-dependent part of enthalpy flows toward the warm end of the regenerator because $\dot{m}$ and $P - P_0$ are roughly in phase (pressure is generally lower when flow is toward the warm end). For the case $T\beta < 1$, the pressure-dependent enthalpy flow is toward the cold end.

In the *stage 3* regenerator, at a mean pressure of 2 MPa, the above plot indicates that the pressure-dependent enthalpy flows toward the warm end above 9.5 K and toward the cold end below 9.5 K. Since the total enthalpy flow is constant within the regenerator (barring any heat flow through the walls) the temperature-dependent enthalpy flow must be higher near the warm end and lower near the cold end. Since the temperature-dependent enthalpy scales roughly in proportion to the temperature gradient, the temperature gradient is higher near the warm end and lower near the cold end, as the plot shows.

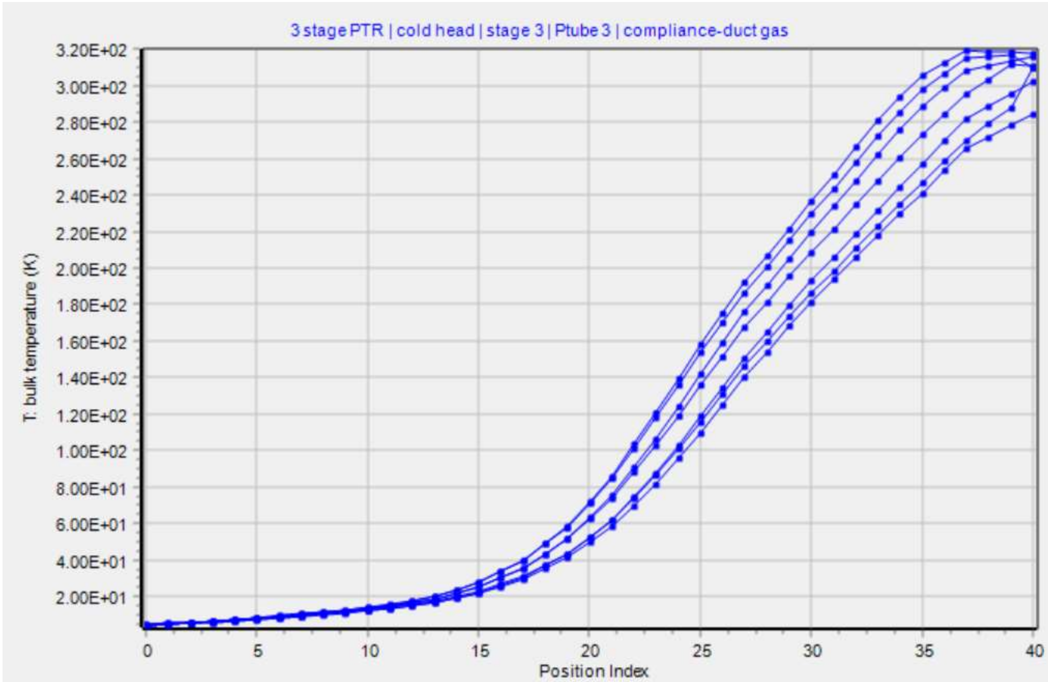
Pressure-dependent enthalpy flow is easy to formulate in terms of the amplitudes and phases for mass flow rate and pressure variation, which are available as model outputs:

$$\dot{H}_T = C_T \langle \dot{m}(P - P_0) \rangle \approx \frac{1}{2} C_T |\dot{m}_1| \left| \dot{P}_1 \right| \cos\theta$$

In the final expression on the right, $|\dot{m}_1|$ is the amplitude of the mass flow rate first harmonic, $|P_1|$ is the amplitude of the pressure first harmonic and θ is the phase difference between them.

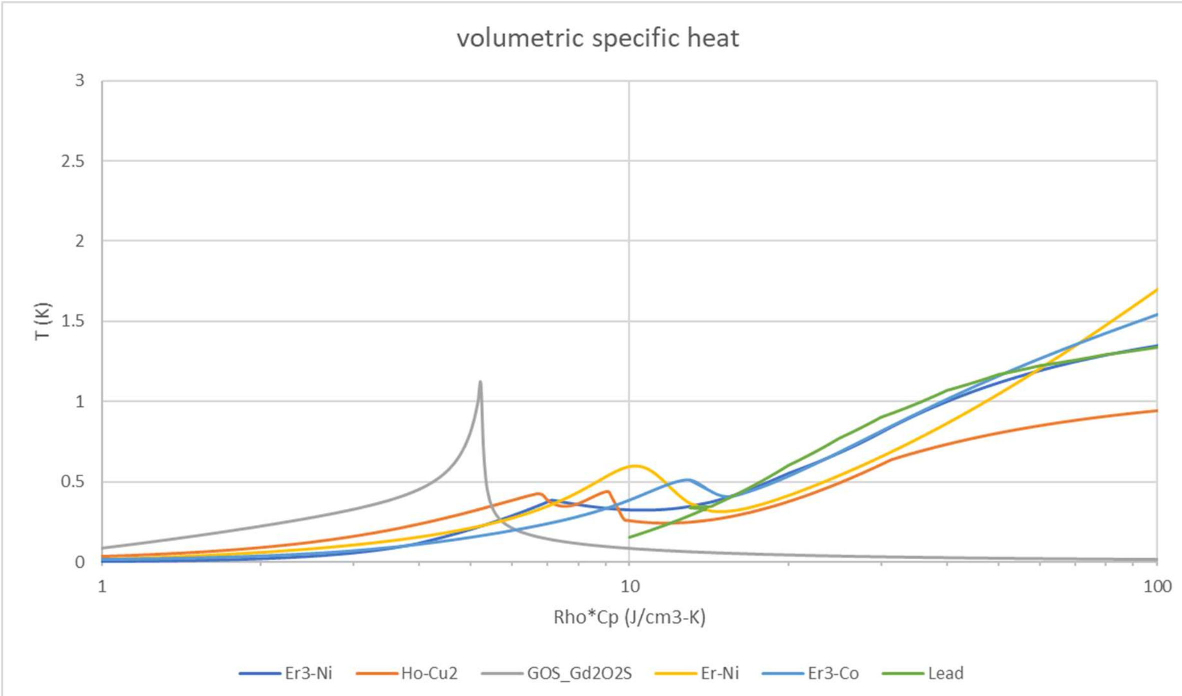
The pressure-dependent enthalpy flow enables another possibility for the *stage 3* regenerator. An intermediate heat exchanger inserted somewhere around the 9.5 K point could provide cooling at an intermediate temperature. Normally there is not a heat exchanger between two regenerators unless there is some duct provided to extract PV power from the helium to balance the cooling power. In this case the pressure-dependent component of enthalpy flow can substitute for the PV power.

Pulse tube effects Since the direction of pressure-dependent enthalpy flow is keyed to the direction of the flow rather than the temperature gradient it flows *toward* the 9.5 K point from both warm and cold ends in *ptube 3*, instead of away. There is a subtlety though in the case of the pulse tube. At least for the limiting case of a perfectly adiabatic pulse-tube, where the helium within is merely a passive transmitter of PV work, moving back and forth with the flow like a compliant displacer. From the point of view of a gas element moving with the flow there is a cycling up and down of pressure and temperature but no heat transfer through the element boundaries. Such a process is governed by the equation of state, which is completely reversible without energy dissipation. So to the extent the pulse-tube is adiabatic there is no thermal effect of the pressure-dependent enthalpy in the pulse tube. Nonetheless, the actual *stage 3* pulse tube temperature profile looks like this:



DC flow effects The *stage 3* cooling power can be increased if the time-average mass flow rate is slightly positive (from the warm to cold end of the regenerator). In the Sage model such a DC flow increases the net enthalpy flow down the regenerator slightly (a loss) but increases the enthalpy flow up the pulse tube significantly (a gain). The result is a net increase in cooling power of the cold heat exchanger (*accept 3)*. The reason for the increased pulse-tube enthalpy flow seems to be that the thermal component of enthalpy flow in the negative direction (thermal loss) is diminished because the temperature gradient is flattened toward the cold end as a result of the DC flow, while the PV power flow in the positive direction is not affected. For the regenerator, the reason that the enthalpy flow does not increase much may be that the temperature gradient does not increase significantly as a result of the DC flow, possibly because of the effects of pressure-dependent enthalpy active in the interior part of the regenerator.

Regenerator materials Low specific heat for regenerator matrix materials is another issue at low temperatures. If the heat capacity relative to the helium heat capacity is too low the regenerator starts to look more like a thermal buffer tube (pulse tube) and loses its ability to suppress enthalpy flow losses. The low-T regenerator material options shown below are available for the regenerator matrix solid (rigorous surface component within packed spheres component). The current model uses Ho-Cu2 for *regen 3*, which has a relatively high heat capacity over the entire temperature range from 4 to 25 K.



From NIST data
https://trc.nist.gov/cryogenics/materials/RegeneratorMaterials/Regenerator%20Materials%20rev%2009-22-06.htm

User-Defined Inputs and Outputs

Each stage has a number of user-defined inputs:

```
Dreg              regenerator ID (m)                    5.895E-02
D0ptb             ptube ID neg bnd (m)                  3.005E-02
D1ptb             ptube ID pos bnd (m)                  3.537E-02
```

Several input dimensions of the components within that stage are recast in terms of these user-defined inputs.

The root-level model contains the following summary outputs for the PV power input to the *pressure wave generator* and the net cooling powers at the various temperature sources:

```
Wpv               pv power input                        4.993E+02
  Wpis
Qlift1            first stage heat lift                 4.976E+00
  QT1
Qlift2            second stage heat lift                4.930E-01
  QT2
Qlift3            third stage heat lift                 3.019E-01
  QT3
```

Optimization

There are quite a few optimized variables for this model. The objective is to maximize the 4 K cooling power (Qlift3) subject to 500 W PV power delivered by the constrained piston to compression space of the pressure wave generator (Wpv) and some arbitrary lower limits for cooling powers at the other stages.

Optimized are:

- Piston diameter for the pressure wave generator (amplitude fixed at 5.0 mm) with compression space mean volume adjusted to maintain a 50% clearance volume at *top dead center*.
- Bypass orifice length and DC flow setpoint (*DCRhoUA*)
- Interstage temperatures
- Regenerator diameters and lengths
- Regenerator porosity or particle diameter, depending on the type of regenerator
- Pulse-tube diameters and lengths (assuming linear diameter taper)
- Inertance tube diameters and lengths

Most heat exchanger lengths are fixed at convenient values except for the main rejector length, which is optimized. Heat exchangers are all modeled as copper screens with heat flow to the outer perimeter.