Sage Model Notes

Cassandra.scfn

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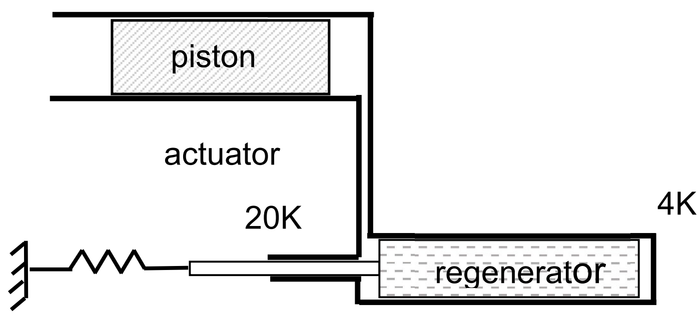
Cassandra

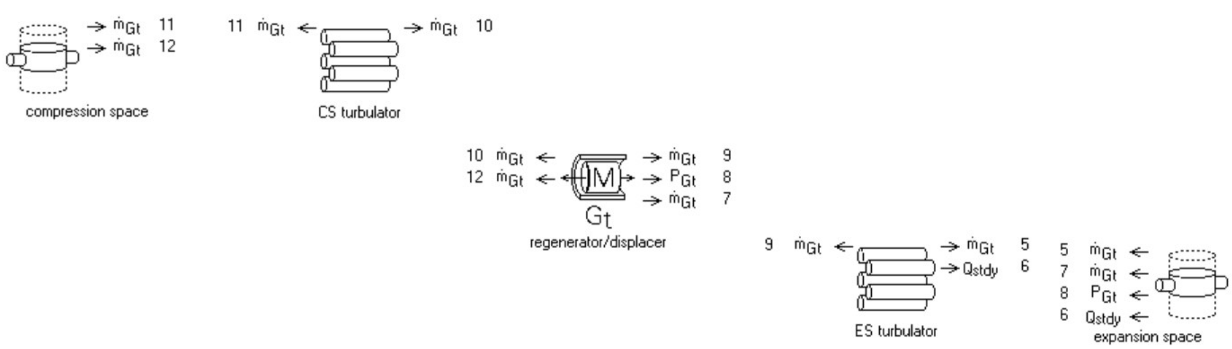
> *Daughter of King Priam and Queen Hecuba of Troy. Given by Appollo the gift of prophecy, but cursed to never be believed.*
>
> — *Greek Mythology*

This model presumes a 20 K pre-cooler (i.e. 20 K heat rejection temperature), then cools down to 4 K with 100 mW cooling power, consuming 1.1 W of PV power, and rejecting 1.0 W of heat. Cassandra predicts that a cooler like this will one day end the reign of the 4 K GM cooler.

For conceptual illustration purposes this stirling-cycle cooler might take the form of a split-cycle configuration, as shown in the following schematic.



The cold-head part of the Sage model consists of a regenerative displacer capped by turbulator holes at each end that lead to a 20 K compression space and 4 K expansion space. Warmer components are at the top and colder components at the bottom in the view below:



According to the model:

Cold Temperature	4 K
Heat Rejection Temperature	20 K
Operating frequency	5 Hz
Charge pressure	40 kPa (sub-atmospheric)
Regenerator diameter	28 mm
Regenerator length	130 mm
Regenerator moving amplitude	5.8 mm
Actuator piston volumetric amplitude	19 cm^3
Actuator PV power	1.08 W
Heat lift at 4 K	0.10 W
Heat rejection 10 K	1.18W

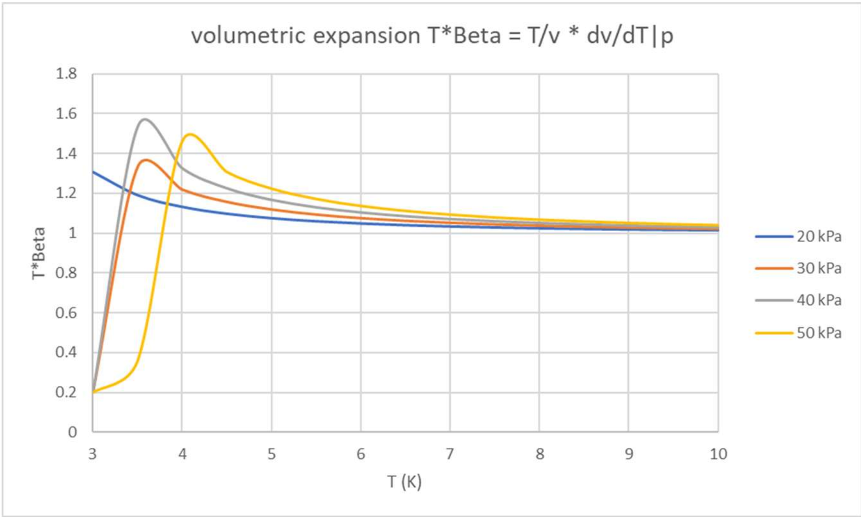
Small lightweight linear actuators with 1 W power ratings are already available for use in the IR sensor cooling industry. There are no fundamental problems to operating linear actuators at 20 K. Electrical efficiencies of 90% are common at room temperature with significant improvements expected at 20 K due to lower coil electrical losses (lower electrical resistance). Superconducting coils are possible. Simple clearance seals using low-wear materials can last several years given low side loads. Simple jet-impingement heat exchangers at the cold tip and heat-rejection ends are entirely adequate. A lightweight thin shell is all that is needed for the pressure vessel — more like a vacuum container.

Benefits of Low Helium Pressure

Commercial free-piston stirling cryocoolers designed for 30–80 K typically operate at helium charge pressures on the order of 1 MPa to achieve sufficiently-high power density, consistent with PV and thermal power flows on the order of 100 W. But for a cryocooler operating in the range 4 – 20 K low pressure makes sense because PV and heat-flows are much lower — on the order of 1 W. There are other advantages to low charge pressure when operating near 4 K, namely:

- Low volumetric helium heat capacity better matches the vanishing heat capacity of regenerator solid materials.
- That, combined with high frequency operation, eliminates large regenerator gas and solid temperature swings.
- Helium behaves more like an ideal gas, instead of like an incompressible fluid at high pressures. More on that below.

Some non-ideal behavior remains at low pressures. One way this shows up is in the volumetric expansion property, designated $T\beta$ in thermodynamic textbooks, which gives rise to a pressure-dependent enthalpy flow akin to the latent-heat enthalpy transport of vapor-compression refrigeration cycles.



For the case $T\beta > 1$ the time-average pressure-dependent part of enthalpy flows from the cold end toward the warm end of the regenerator boosting cooling power. It is difficult to make use of this effect in pulse-tube refrigerators because what flows up the regenerator flows down the pulse tube, from hot to cold end, canceling the effect. But in stirling refrigerators there *is* no pulse tube so the pressure-dependent enthalpy flow can be fully exploited.

User-Defined Variables

The root model contains some convenient user-defined inputs used to recast other model inputs.

```
Inputs
  TPrecool          precool temperature (K)                    2.000E+01
  Tcold             cold-end temperature (K)                   4.000E+00
  XpAmp             piston amplitude (m)                       9.951E-03
  XdAmp             displacer amplitude (m)                    5.823E-03
  XdPhase           Xd phase (deg)                             4.857E+01
  Dpis              piston diameter (m)                        4.000E-02
  IDpwall           regenerator pressure wall ID (m)           2.816E-02
  Drod              drive rod diameter (m)                     1.000E-03
  Lregen            regenerator length (m)                     1.305E-01
  GapSeal           clearance seal gap (m)                     1.000E-05
  Eccen             eccentricity seal 0 to 1 (NonDim)          5.000E-01
  GapAppendix       appendix gap (m)                           3.000E-04
```

Also, a number of user-define outputs some of which are used to recast model inputs and others summarizing thermodynamic performance.

```
Outputs
  Adis              displacer frontal area                     6.226E-04
    0.25 * Pi * Sqr(IDpwall)
  Apis              piston frontal area                        1.257E-03
    0.25 * Pi * Sqr(Dpis)
  Arod              drive rod frontal area                     7.854E-07
    0.25 * Pi * Sqr(Drod)
  Wpv               total piston PV power                      -1.078E+00
    Wdis + Wpis
  Qin               net heat input                             1.000E-01
    Qes + QparaSource
  Qrej              net heat rejection                         1.178E+00
```

```
    Qduct + Qcs2 + Qseal + QparaSink
  COP               coef of performance                     9.280E-02
    -Qin / Wpv
  PmaxCS            peak pressure CSpace                    5.000E+04
    Pmax
  WallRegen         regen pressure wall thickness           2.500E-04
    max(WallMin, PmaxCS*IDPwall/(2*AllowedStress))
  EccenFac          effective eccentric gap ratio           1.112E+00
    Power(1 + 1.5*Sqr(Eccen), 1/3)
```

Optimization

The model contains an optimization specification to minimize heat rejection (pre-cooler load) subject to 100 mW of 4 K cooling:

```
Objective Function
  Minimize Qrej

OPTIMIZED VARIABLES                  SUBJECT TO CONSTRAINTS

SCFusion model
  TPrecool                           Qin = 0.10
  XpAmp                              Tprecool = 20
  XdAmp                              PmaxCS < = 5.0E4
  XdPhase                            Freq > = 5
  IDpwall
  Lregen
  Freq

1 pressure source
  Pcharge

8.2 CS turbulator
  Dtube

8.3.2.1 packed sphere matrix
  Dsphere
```

The point is to provide enough maneuvering room (optimized variables) for Sage to find an optimal configuration without getting bogged down in details. For example, practically nothing about the compressor is modeled apart from its swept volume, by way of piston amplitude for a fixed cylinder diameter.

Frequency is constrained above 5 Hz with the idea that this is supposed to be a "high-frequency" cooler. COP tends to increase at lower frequency, but so does size.

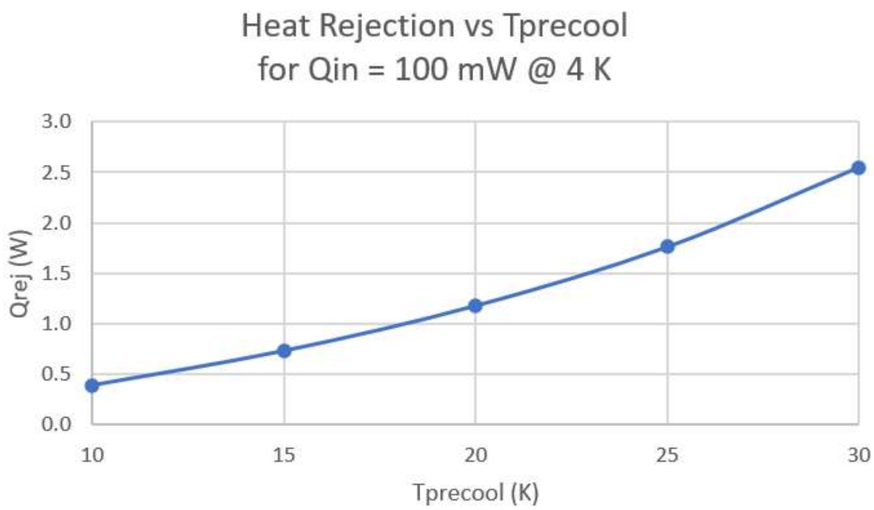
Charge pressure is limited by the constraint `PmaxCS < = 5.0E4` The reason for this is to avoid helium liquefying at the cold end, which tends to mess up convergence and in a practical sense would block the regenerator. The helium gas-liquid saturation pressure is 0.82 bar at 4 K. Helium liquefaction at 4 K is also a potential issue in high-pressure GM coolers, but to avoid this they operate in a supercritical state above the critical pressure of 2.28 bar. Only Joule-Thomson coolers seem to thrive in the saturation zone between 0.82 and 2.28 bar.

The precooler temperature `Tprecool` is itself optimized subject to an equality constraint, which is a convenient way to change the pre-cooler temperature simply by changing the constraint and re-running the optimization. That way the optimizer can decide to change `Tprecool` in gradual steps if it gets into convergence trouble.

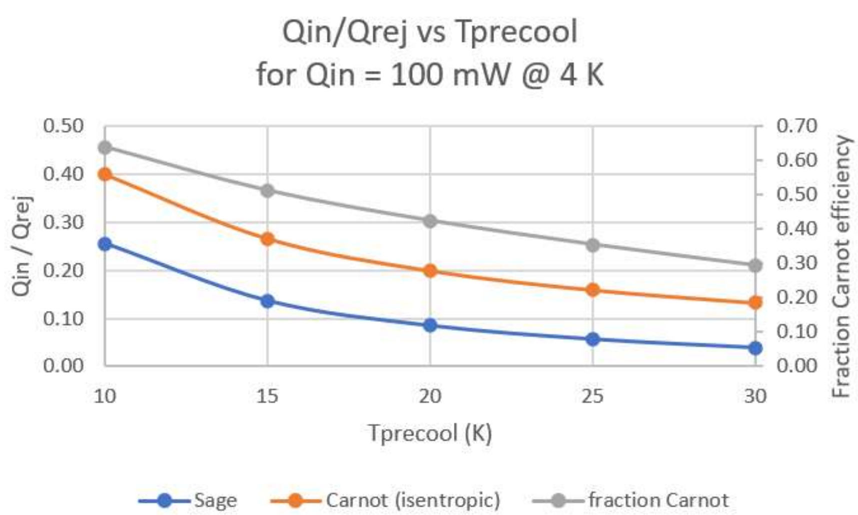
Precooler Temperature Map

The following plots show the result of optimizing the model over a number of pre-cooler temperatures in the range of 10 – 30 K.

The first plot shows the precooler cooling load required for 100 mW cooling at 4 K.



The second plot shows the thermodynamic efficiency, including the efficiency relative to an ideal (isentropic) cooling cycle.



The fraction of Carnot efficiency is relatively high at the 10 K end but decreases with increasing temperature ratio as it inevitably does.

Precooler Options

Commercial GM coolers can easily cool to the range of 10 – 30 K, but are rather large and complicated. Commercial free-piston linear-motor cryocoolers typically bottom out around 30 – 40 K. To use a free-piston cooler as a precooler for a Cassandra 4 K cooler might require an intermediate stage. It too could be a relatively low-pressure, high-frequency, lightweight device. The two stages might be distinct self-contained stages, each independently optimized, coupled by thermal links. Depending on the amount of 4 K cooling required, the overall system could be significantly smaller, cheaper and less intrusive than a GM cryocooler. So says Cassandra.