Sage Model Notes

SolarPanPipe.scfn

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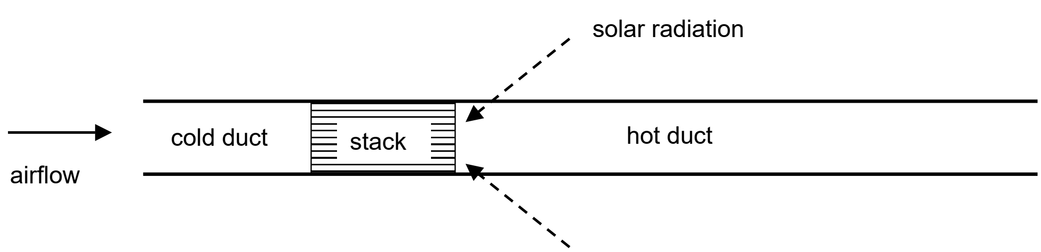
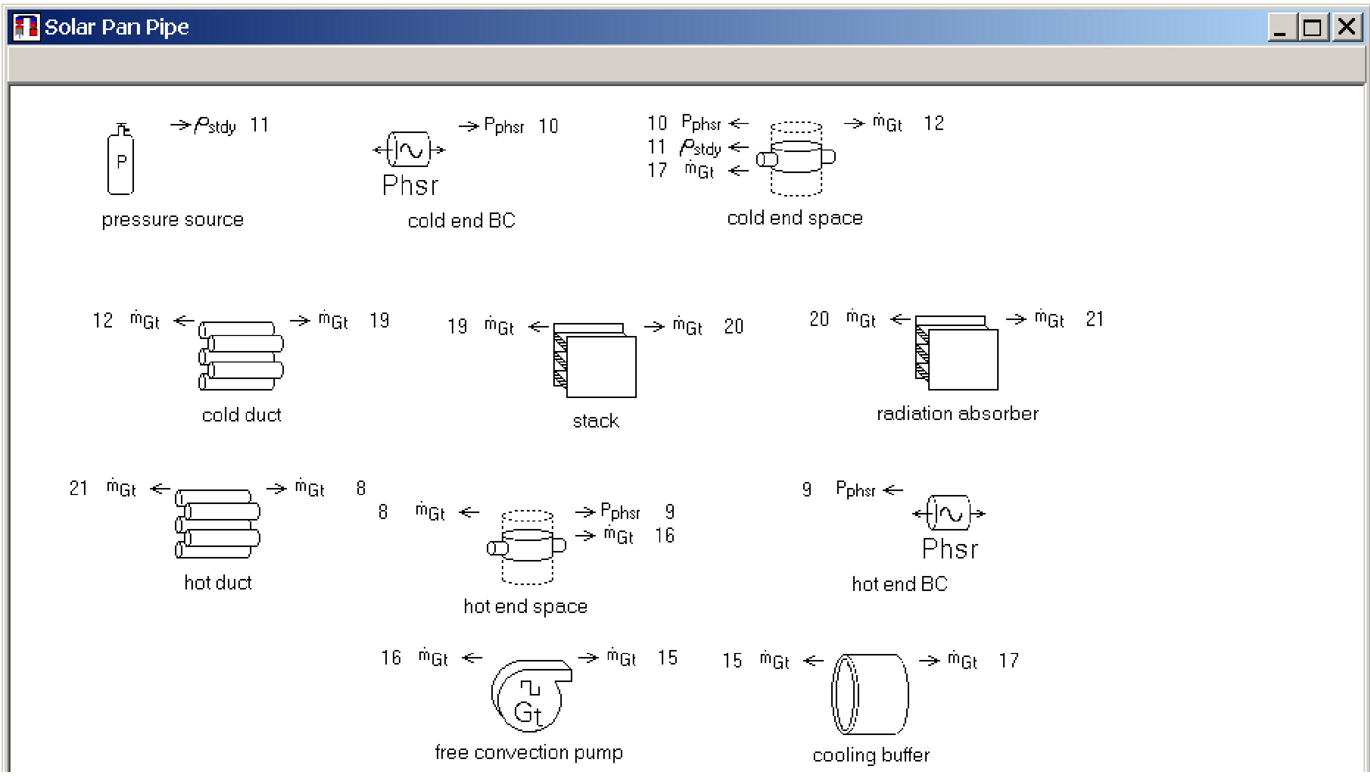
A model of a thermoacoustic singing-tube, open at both ends with the stack actively heated at one end and cooled at the other by through air flow. The basis for this model was a singing tube made in the 1990's by John Corey of CFIC Inc., where the stack heating was accomplished by focused solar radiation through transparent tube walls — hence the model name SolarPanPipe. The cooling air flow was induced by free convection with the hot end of the stack oriented upward.The idea likely originated in earlier work by John Wheatley and Greg Swift at Los Alamos National Laboratory and before that in the writings of Lord Rayleigh. I myself once made such a machine that heated the stack by a propane flame burning at the hot end of the stack. Here is a schematic of the physical layout:



In the Sage model the components are arranged in order of occurrence with cold parts at the top of the edit window and hot parts at the bottom:



Components upstream of the *cold duct* and downstream of the *hot duct* are fictitious (non-physical) components required to implement the acoustic boundary conditions at the open ends and the cooling flow. The pistons attached to the gas spaces actually drive the acoustics within the ducts, but Sage's optimizer adjusts their amplitudes and relative phase so as to be consistent with the required acoustic impedance boundary conditions of open-ended tubes. More on that below. The *free convection pump* component supplies a dc flow through the device, cooled by the *cooling buffer* heat exchanger before entering the *cold duct*. The acoustical power output is just the mechanical power delivered to the two pistons once their motions are adjusted to satisfy the open-end impedance boundary conditions.

The spacing between the plates in the *stack* is of the order of the thermal diffusion length in the gas, so accurate modeling requires the complex Nusselt number formulation. Therefore, the stack is modeled as rectangular channels, which employ the complex Nusselt number formulation, rather than as a foil matrix, which employs the real-valued low Valensi number limit.

Open End Impedance

The variable volume spaces at the duct open ends represent the atmosphere immediately outside the ducts. The volumetric displacements of the two pistons represent the displacements of the radiated sound to parts beyond. The phasing of one of these pistons is arbitrary (*cold end BC* in this case) but the amplitude and phase of the other must be set to satisfy the acoustic conditions at the tube ends. The operating frequency must also be set to match acoustic conditions, because a thermoacoustic resonator is a self-exciting device that determines its own frequency. So there are four variables to solve for, subject to two acoustic impedance boundary conditions.

Morse and Ingard ([2], section 9.1, pp. 467–474) discuss the issue of impedance in acoustical ducts. Assuming the pressure and velocity at the desired point of interest in a plane standing are given by the complex formulation $\boldsymbol{p}e^{-i\omega t}$ and $\boldsymbol{u}e^{-i\omega t}$, the impedance is just the ratio of the complex amplitudes $\boldsymbol{p}/\boldsymbol{u}$. They put this in the form

$$\boldsymbol{p}/\boldsymbol{u} = \rho c(\theta - i\chi) \tag{1}$$

and then show some curves in figure 9.2, derived from [3], on the basis of which the following approximations appear reasonable at the open end of a tube of diameter D for a wave of wavelength λ

$$\theta \approx \frac{\pi^2}{4}(D/\lambda)^2 \tag{2}$$

$$\chi \approx 1.927D/\lambda \tag{3}$$

These approximations hold only for $D/\lambda < 1/10$. As D/λ approaches zero, these impedance values serve to reflect most of the sound back into the tube with a pressure node just outside the tube end. Consequently, long narrow tubes radiate sound poorly.

As modeled by Sage, the acoustic impedance at an open end is formulated in terms of the pressure and boundary mass flow rate of the variable volume space to which the open tube end is attached. Complex pressure amplitude $\boldsymbol{p}$ is just $p_c + ip_s$ where p_c and p_s are the cosine and sine coefficients of the first harmonic of the FPMean Fourier series. Velocity amplitude $\boldsymbol{u}$ is

$(1/\rho A)(\dot{m}_c + i\dot{m}_s)$ where $\dot{m}_c$ and $\dot{m}_s$ are the cosine and sine coefficients of the first harmonic of the FRhoUANeg Fourier series. A velocity Fourier series is not directly available. So the impedance at the open end is just

$$\boldsymbol{p}/\boldsymbol{u} = \frac{\rho A}{\dot{m}^2}\left[(p_c\dot{m}_c + p_s\dot{m}_s) - i(p_c\dot{m}_s - p_s\dot{m}_c)\right] \tag{4}$$

Equating the right sides of equations (1) and (4), substituting the above approximations for θ and χ, then $\pi D^2/4$ for A and c/f for λ and simplifying gives the final form of the open-end impedance constraints used in the model:

$$\frac{c}{f^2\dot{m}^2}(p_c\dot{m}_c + p_s\dot{m}_s) = \pi \tag{5}$$

$$\frac{D}{f\dot{m}^2}(p_c\dot{m}_s - p_s\dot{m}_c) = 2.45 \tag{6}$$

Optimization

User-defined variables in the *cold end space* and *hot end space* implement the above impedance constraints and Sage's optimizer solves both piston amplitudes Xamp and hot piston phase Xphase in order to satisfy the constraints.

In addition to that the model is set up to optimize the lengths (Length) of the cold and hot ducts, the *stack* dimensions (Wchan, Nchan, Length) and *radiation absorber* heat exchanger dimensions (Wchan, Nchan), recasting Hchan inputs to equal Wchan for both. A few geometrical constraints require the open area of the *stack* and *radiation absorber* equal to the duct open area. Sage's optimizer also adjusts the cooling flow mass flow rate (FRhoUA.Mean of the free convection pump) with the objective of maximizing the acoustical power output(user variable Wrad).

Convergence

This is a highly nonlinear model that does not converge as reliably as simpler models. Convergence may fail after significant changes to certain input values or attempting to solve from a re-initialized state. If this happens one strategy is to revert to a previously converged model and make smaller changes in succession. If that is not possible then try temporarily reducing the FRhoUA.Mean input of the *free convection pump* component or the Xamp values of the *cold end BC* and *hot end BC* piston component. Then gradually increase them again after successful convergence.

References

[1] G. W. Swift, *Thermoacoustic Engines*, J. Acoust. Soc. Am., **84**(4), October 1988

[2] P.M. Morse & K. U. Ingard, Theoretical Acoustics, McGraw-Hill, (1968)

[3] H. Levine and J. Schwinger, *On the Radiation of Sound from an Unflanged Circular Pipe*, Phys. Rev. **73**:383, (1948)