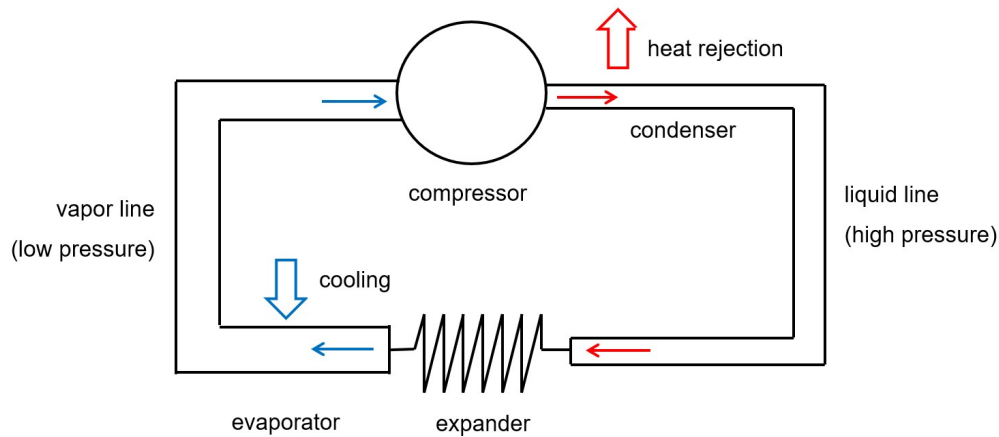


Sage Model Notes

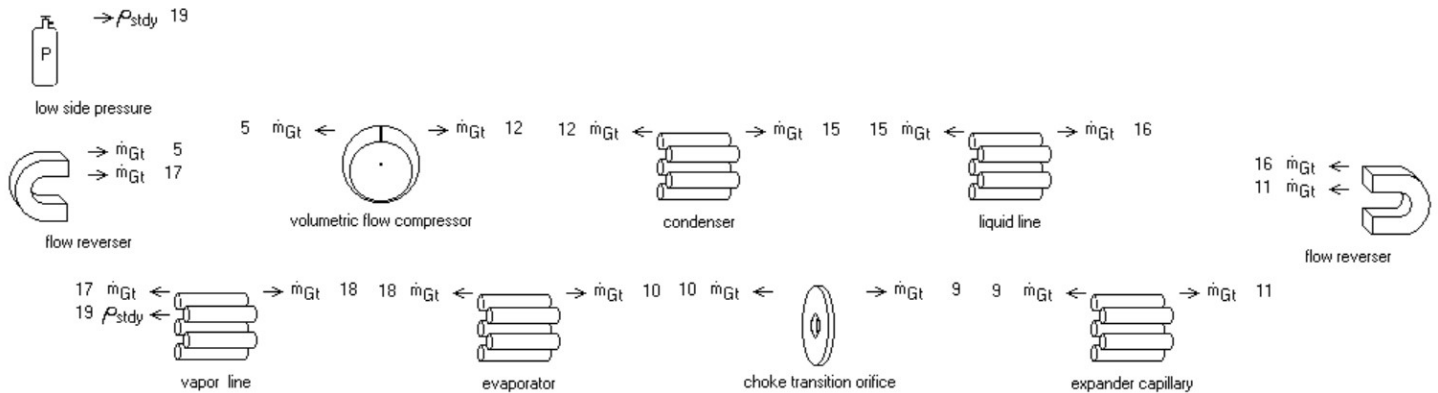
RefrigerationLoop.scfn

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A model of a vapor-compression refrigeration cycle, where a high-pressure liquid refrigerant at 40 C at the exit of a condenser heat exchanger expands within a capillary tube to a lower pressure and subsequently vaporises beginning near 0 C at the entrance of an evaporator to provide cooling. This model could serve as a starting point for modeling a refrigerator (box for cooling food), an air conditioner or a heat pump.



The Sage model looks like this:

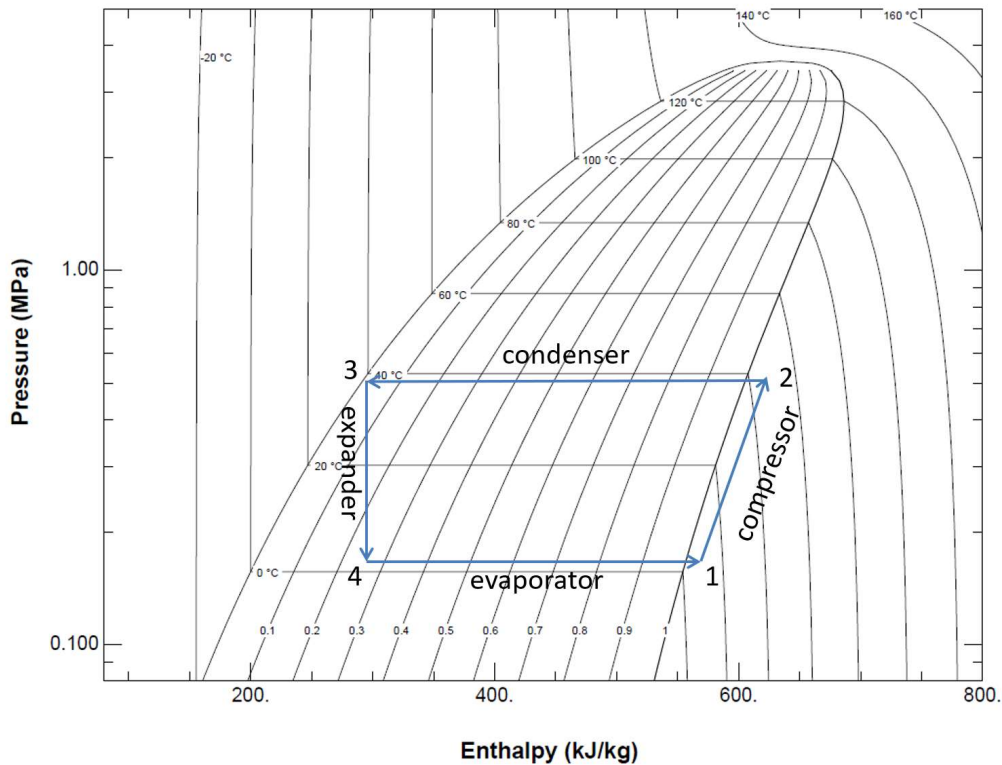


Refrigerant flows in a clockwise loop, generally to the right through high pressure components in the top row, returning to the left through low pressure components in the bottom row. The *low side pressure* bottle imposes a low-pressure boundary condition at the negative end of the *vapor line*, just upstream of the *volumetric flow compressor*, set according to the saturation pressure for the refrigerant at the desired evaporator temperature. In this case the refrigerant (Gas input) is isobutane (R 600a) with a saturation pressure of 1.57 bar at 0 C. The *volumetric flow compressor* provides a positive (directed toward right) flow with the volumetric flow rate input chosen to produce

a pressure-rise to the saturation pressure at the condenser exit temperature, in this case 6.35 bar at 50 C.

On the pressure-enthalpy diagram below process 4-1 takes place in the *evaporator*, process 1-2 in the *volumetric flow compressor*, process 2-3 in the *condenser* and process 3-4 in the *expander capillary* and *choke transition orifice*. The compression process takes place entirely in the vapor phase just beyond the dew line of the two-phase region. The expansion process takes place in the two-phase region starting from a vapor quality (curves within the two-phase region) near $X = 0$ (0% vapor by mass) at the expander entrance to $X = 0.27$ (27% vapor by mass) at the evaporator entrance. That does not sound like mostly vapor, but the density of the liquid (580 kg/m^3) is about 130 times higher than the vapor (4.3 kg/m^3) so the vapor is more than 99% of the total fluid volume. Volume-wise, it is mostly vapor exiting the expansion capillary. The remaining evaporation to a quality $X = 1$ (100% vapor) takes place in the evaporator.

5: Pressure vs. Enthalpy plot: isobutane



The math behind the quality-volume relationship starts with the definition of vapor quality as the fraction of vapor mass to total mass (vapor + liquid):

$$X = \frac{m_V}{m_V + m_L}$$

So,

$$\frac{1}{X} = 1 + \frac{m_L}{m_V} = 1 + \frac{\rho_L V_L}{\rho_V V_V}$$

And after some algebra

$$\frac{V_V}{V_L} = \frac{\rho_L}{\rho_V} \frac{X}{1-X}$$

Root Model

The root model contains a number of user-defined inputs and outputs:

Inputs

Tevap	evaporator temperature (C)	1.000E+01
Tcond	condenser temperature (C)	4.000E+01
Dcapillary	capillary ID (m)	9.835E-04
HmultBoiling	heat transfer multiplier (NonDim)	1.000E+01
HmultCondensing	heat transfer multiplier (NonDim)	1.000E+01

Tevap and Tcond set the exit temperatures of the evaporator and condenser (see Convergence section below). Dcapillary adjusts the diameters of the *expander capillary* and *choke orifice* which together impose the throttling pressure drop of the refrigeration cycle and indirectly the pressure rise of the volumetric flow compressor. HmultBoiling and HmultCondensing set empirical Hmult heat-transfer multipliers (see Boiling and Condensing Transport Properties below).

Outputs

Win	mechanical input power	5.000E+01
-Wcomp		
Qlift	cooling power	2.133E+02
Qevap		
Qrej	heat rejection	-2.633E+02
Qcond		

Win gives the net mechanical input power to the volumetric flow compressor. Qlift and Qrej give the net cooling power and heat rejection through the walls of the *evaporator* and *condenser*. The sum of these outputs is zero according to an energy balance principle. Electrical power input is not defined since there is no electrical motor component driving the compressor in the model.

Volumetric Flow Compressor

This component does not model any particular type of compressor. Rather it represents a high level abstraction that characterizes a compressor according to these inputs:

Efficiency	adiabatic efficiency (NonDim)	8.000E-01
Rclearance	relative clearance volume	1.000E-02
Vdot	volumetric flow rate (m3/s)	1.846E-04

Whatever the actual compressor, Vdot represents the compressor volumetric displacement times the frequency, Rclearance represents the ratio of gas clearance volume at the end of the compression process relative to the volume at the beginning, and Efficiency represent the ratio of ideal adiabatic compressor power input relative to the required mechanical input power. The actual mechanical input is generally higher because of leakage losses and heat transfer out of the gas. As Rclearance increases it reduces the maximum pressure at the compressor exit, like a real compressor. More details are found in the Sage User's Guide.

Boiling and condensing heat transfer

Sage does not directly understand heat transfer or flow resistance in tubes with mixed two-phase flow. Instead, root-level inputs `HmultBoiling` and `HmultCondensing` set the empirical `Hmult` heat-transfer multipliers in the *evaporator gas* and *condenser gas*. Both multiplier factors are 10 in this model, which means there will be 10 times more heat transfer for a given fluid-wall temperature difference than for a single-phase fluid with the same density and flow velocity. These values might be adjusted in a more evolved model based on known correlations for boiling and condensing heat transfer for the actual flow conditions. Likewise for flow-friction in two-phase flow. User-defined input to set `Fmult` in the *evaporator gas* and *condenser gas* might be also make sense in a more evolved model.

Vapor Quality Limits

In this model the vapor quality is close to zero (liquid state) at the exit of the condenser, i.e. close to the so-called bubble line bounding the two-phase region of the pressure-enthalpy diagram. This produces maximum cooling power for a given condenser temperature and also minimizes the flow resistance through any “*liquid line*” connecting the condenser to the expander. In a practical refrigerator the liquid at the condenser exit may be cooled well below the boiling point by lowering the condenser temperature, to avoid boiling in the liquid transfer line.

The Sage model has some trouble converging as the fluid state in the condenser approaches the bubble line because of the extremely nonlinear rate of change of fluid density as a function of pressure and temperature there. If it is necessary to model sub-cooled liquid exiting the condenser it often helps to gradually increase the compressor flow input `Vdot` or reduce the condenser temperature input `Tcond` until well into the liquid phase.

It is possible to monitor vapor quality in most of the gas domains of this model via these user defined outputs:

```
QualNeg          vapor quality neg bnd          1.000E+00
  Gas.Qual(RhomNeg, TNeg)
QualPos          vapor quality pos bnd        1.853E-03
  Gas.Qual(RhomPos, TPos)
```

The above values apply to the condenser and show that the fluid enters in the pure vapor state (slightly superheated, actually) and leaves near the liquid line, but not completely liquified (see Optimization section below).

These outputs reference the working gas quality property, available via the `Gas.Qual` function with fluid density and temperature arguments. Variable `TNeg` and `TPos` (endpoint mean fluid temperatures) have been built into Sage gas domains for a long time. `RhomNeg` and `RhomPos` (mean densities) were introduced in Sage version 13.

Choked Flow

The addition of a sharp-edged orifice at the low-pressure exit of the capillary is an aid solution convergence. Without the orifice, the capillary flow velocity may accelerate to the sonic velocity at the exit and the pressure cannot decrease further to the low side pressure — the flow is choked. The sharp-edged orifice is the only Sage component capable of dealing with choked flow. It imposes a pressure drop into the solution, corresponding to the shock-boundary expansion of the sonic flow leaving the orifice at the capillary exit pressure. In the present model the pressure at the capillary exit is 2.56

bar and the orifice reduces the pressure only slightly to 1.73, possibly because this is an optimized model. With different model inputs the orifice can be much higher and the choked-transition orifice more essential.

Refrigerant Fluid

The working fluid of the model (Gas input) is Isobutane (R600A), which is option `refprop isobutane` in the standard GasSCF.dta data file. If you want to investigate a refrigerant not in the standard data file you will have to use the RefpropToSage utility to create a Sage property file (*.dta) from one of the refrigerants in the Refprop data base. There are so many possible refrigerants available that it is not practical to include each one in the GasSCF.dta file. You can then use the PropBase utility to add your refrigerant into the GasSCF.dta file or use the Options | Model Class dialog to select your individual refrigerant file as the gas property file.

Optimization

This model contains a relatively simple optimization that sizes some of the components:

Objective Function

Maximize cooling power (Qlift).

Optimized Variables

Description	Sage variable name
Compressor volumetric flow rate	Vdot
Expander and orifice common diameter	Dcapillary
Low-side pressure	Pcharge
Condenser tube diameter	Dtube
Evaporator tube diameter	Dtube

Optimizing Vdot provides slack to meet the power input constraint. Optimizing Dcapillary allows the compressor to adjust to satisfy the condenser temperature setpoint. Optimizing Pcharge provides slack to satisfy the evaporator temperature setpoint. Optimizing condenser and evaporator Dtube provide slack to maximize the objective function.

Constraints

Description	Sage variable name
Mechanical power input = 50 W	Win

Maximizing cooling power for a fixed mechanical power input automatically forces the vapor quality at the condenser exit to approach zero.

Convergence

This model may be prone to convergence difficulty, especially when starting from an initialized state. Temporarily reducing the magnitude of the Vdot input of the *volumetric flow compressor* should help. Another option is to adjust the normalization values of the root model:

Lnorm	length scale (m)	5.000E-03
FreqNorm	frequency scale (Hz)	6.000E+01
Pnorm	pressure scale (Pa)	1.100E+05

Pnorm defines the initial pressure of the solution and the above value is the low-side pressure (set by low-pressure sink), which should allow the pressure of the component

downstream of the orifice (the evaporate entrance) to remain steady while the pressure of the upstream components (the condenser exit and expander capillary) gradually rise. Varying $Lnorm$ and $FreqNorm$ may help re-normalize the model in case the solution appears singular initially. For many models $Lnorm$ in the range 0.005 – 0.01 and $FreqNorm$ in the range 50 – 100 Hz seem to work best.

This being a steady-state model ($NTnode = 1$) the frequency input $Freq$ is irrelevant. But $FreqNorm$ is used to normalize solution variables like mass flow rate, velocity and many other quantities representing rates of change. So it is very relevant.

The initial temperatures of all model components ($Tinit$ values) are set to 20 C, which also helps convergence from an initialized state. The inputs of the *independent line heat source* components inside the condenser and evaporator determine the final solved temperatures. These temperatures are set by recasting the normal inputs in terms of the root-model user-defined temperature inputs $Tcond$ and $Tevap$.

Condenser

```
Tsrc = unit spline...
      (0.000E+00, Tcond + 10)
      (1.000E+00, Tcond)
```

Evaporator

```
Tsrc = unit spline...
      (0.000E+00, Tevap)
      (1.000E+00, Tevap - 10)
```

These values produce a gradual change in the boundary temperature by 10 C to avoid sudden condensation or vaporization events during model development. A more evolved model might adjust the independent heat source boundary temperatures according to the external heat exchangers that reject the heat of condensation and connect the evaporator to a cooled load.

Also potentially helpful is to increase the value of the $UpwndFrac$ input in gas domains (upwind influence for density and temperature interpolated values) above the default value of 0.01. All tubular heat exchanger components in this model use $UpwndFrac = 0.5$, which helps to prevent jagged temperature and density distributions.

To better resolve the abrupt density changes in components like the condenser, evaporator, expander capillary and evaporator, it helps to increase the number of computational cells ($NCell$ input) higher than for single-phase models. In the present model $NCell = 50$ for all these components. The fluid density in the condenser looks like this:

