Sage Model Notes

SpringMassDamper.scfn

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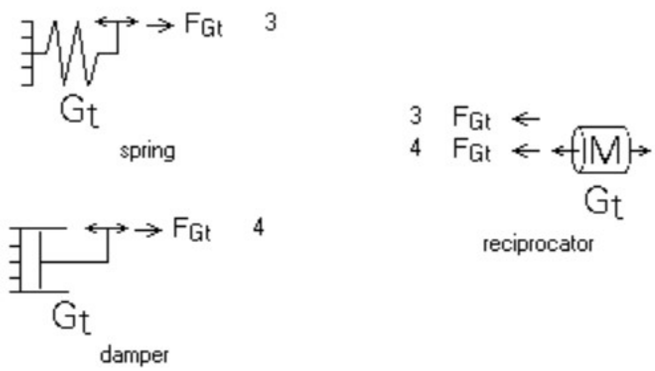
Intended as a tutorial for those just learning Sage. A simple model of a spring-mass-damper linear system driven by a forcing function. The Sage model looks like this in the edit window of the Sage GUI:



Each component contains input variables that define its behavior and output variables that summarize its state after the model is solved. To show these, select each component in turn and click the right mouse button. In the popup context menu select Show In Display Window to display input and output values in the display window to the right of the edit window. Current inputs are:

Spring
```
  K                stiffness (N/m)                            1.000E+04
```

Damper
```
  D                damping coef ((N s)/m)                     1.000E+02
```

reciprocator
```
  Mass             reciprocating mass (kg)                 1.000E+00
  FF               forcing function (N, rad)                  1.000E+01...
    ( 1.000)E+02 Amp
    ( 0.000)E+00 Arg
```

The `FF` input specifies the forcing function acting on the reciprocator. The number 1.000E+01 at the end of the first line is the time-mean value of the forcing function. The next two lines of the display encode the amplitude and phase of the first harmonic (sinusoidal component). The above values re equivalent to a sinusoidal time variation with amplitude 1.000E+02 and phase 0. In mathematical notation the forcing function is

$$\mathrm{F} = 10 + 100cos(\omega\mathrm{t} + 0)$$

The root model inputs are visible in the SCFusion-model tab of the display window:

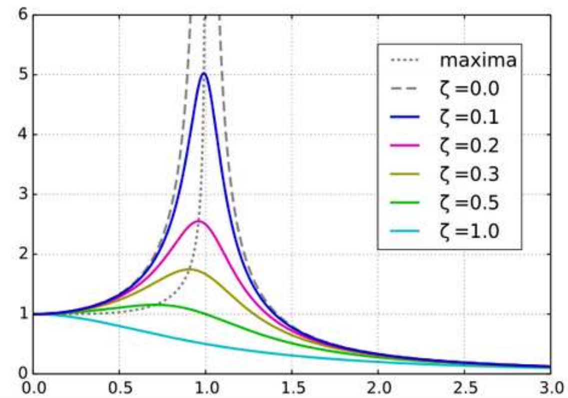
```
  NTnode           number time nodes                          7
```

Lnorm	length scale (m)	1.000E-02
FreqNorm	frequency scale (Hz)	6.000E+01
Pnorm	pressure scale (Pa)	1.000E+06
Tnorm	temperature scale (K)	3.000E+02
Qnorm	heat flow scale (W)	1.000E+02
Vnorm	voltage scale (V)	1.000E+01
Inorm	current scale (A)	1.000E+00
Freq	frequency (Hz)	1.125E+01

Most of these are default values except for Freq which has been optimized (see below) to be the resonant frequency where the piston amplitude response is maximum. The "norm" variables are representative values used to normalize the internal solution variables. There is usually no reason to change these, although can affect convergence in some cases. The Sage User's Guide (online Help) discusses this in more detail.

Model Background

The analysis of spring-mass-damper linear systems, aka simple harmonic oscillators, is well studied in the mechanical engineering literature. As an early example see Den Hartog, Mechanical Vibrations , McGraw-Hill, 1947. Analysis typically focuses on the periodic steady-state solution where a sinusoidal forcing function leads to a relatively straight forward sinusoidal response. Standard results are plots of reciprocator amplitude as functions of frequency at different damping values, such as this plot from Wikipedia:



Steady-state variation of amplitude with relative frequency ω/ω_0 and damping ζ of a driven simple harmonic oscillator

Solving

After setting the inputs select the Process | Solve menu item to *solve* the model. Sage does not implement a time-stepping solution, starting from time zero. Rather it implements an iterative process that updates all dependent variables of the model simultaneously over a full periodic cycle. The approach is similar to that of the above linear-system analysis. Even though the solution happens to be sinusoidal (phasor variables) for this model, Sage can also solve for non-sinusidal periodic solutions.

Outputs

Each model components can have its own output variables. The main output of interest is the response of the reciprocator to the forcing function at the input frequency, given by the output variable

```
FX              displacement (m, rad)                1.000E-03...
  ( 1.155,  0.000,  0.000)E-02 Amp
  (-0.955, -1.144,  3.017)E+00 Arg
```

The number 1.000E-03 at the end of the first line is the time-mean displacement, caused by the time-mean value of the forcing function opposed by the time-mean deflection of the spring. The next two lines display the amplitudes and phases of the first three harmonics in a succinct format. (see Sage User's Guide in online help for details) In the present case there are actually three harmonics available in the solution but the last two are zero because the input forcing function includes only the first harmonic. In mathematical notation the response is

$$X = 0.001 + 0.01155\cos(\omega t - 0.955)$$

For convenient referencing of the first-harmonic response the reciprocator contains two user-defined outputs

```
Xamp             response amplitude                       1.155E-02
  FX.Amp.1
Xarg             response phase                          -9.554E-01
  FX.Arg.1
```

Note of potential interest In the case of NTnode = 1 the time-mean solution is the only one resolved and Sage defaults to a steady-state equilibrium model.

The mechanical power dissipation in the damper is also a Fourier Series output

```
W                boundary power inflow (W, rad)           3.334E+01...
  ( 0.000,  3.334,  0.000)E+01 Amp
  ( 0.000,  1.231,  2.214)E+00 Arg
```

The time-mean value 3.334E+01 is the value of interest in most cases. But there is also a second harmonic present. In mathematical notation

$$W = 33.34 + 33.34\cos(2\omega t + 1.231)$$

The second harmonic arises from the definition of instantaneous power flow as the product of damping force $D\dot{X}$ and velocity $\dot{X}$

$$W = D\dot{X}^2$$

If $X = X_1 cos\omega t$ this becomes

$$W = \frac{D(\omega X_1)^2}{2}(1 + sin2\omega t)$$

In other words the power dissipation is always positive but fluctuates in proportion to $1 + sin\ 2\ \omega t$.

The time-mean damper power dissipation is balanced by an equal and opposite power delivered by the reciprocator

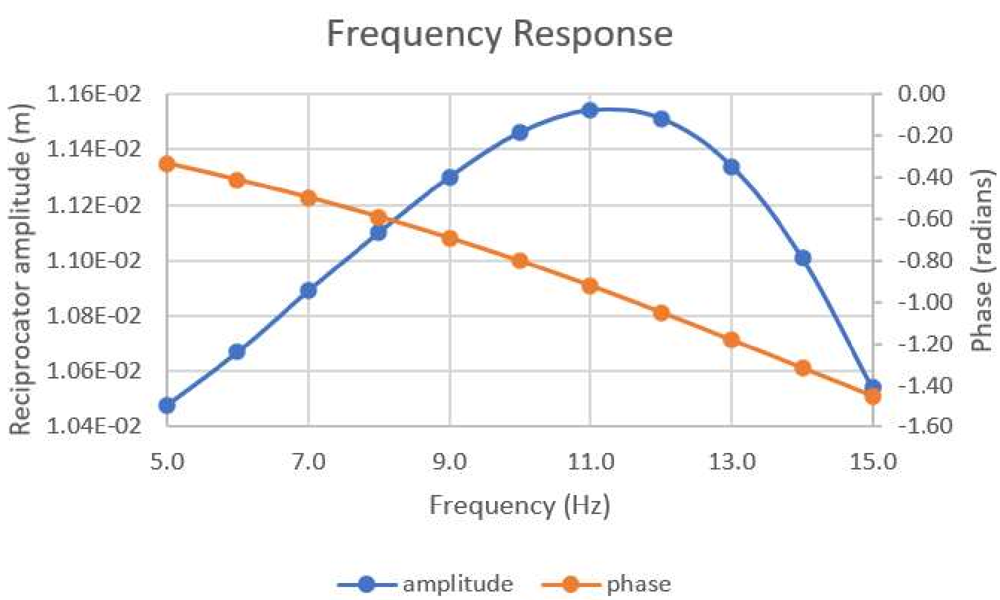
```
W                boundary power inflow (W, rad)          -3.334E+01...
  ( 0.817,  5.774,  0.000)E+01 Amp
  (-2.526, -2.866,  2.459)E+00 Arg
```

Sage solutions always conserve energy, which is especially simple in this model because the damper is the only component that dissipates energy. The spring component absorbs energy during compression and releases it during expansion with a zero time average over the cycle.

Mapping

This model implements a mapping specification that automatically varies the frequency over a range and solve the model at each frequency. Use menu item Specify | Mapped Variables to select variables to be mapped and specify the range and number of values to map. Then use Process | Map to run the mapping.

Similar to the above analytic response plot, the plot below shows the results of mapping frequency over the range of 5 – 15 Hz for a single damping coefficient. Values are logged in a tab-delimited text file, comprising mapped variables and those user-defined outputs with the “write to log file“ box checked. In this case the amplitude and phase of the reciprocator response. Use the Tools | Explore Custom Variables menu item to create and view user-defined variables. The data in the log file can then be plotted under separate software (like MS Excel).



Optimization

This model also implements an optimization specification that automatically optimizes the frequency in order to maximize the response amplitude. Use menu item Specify | Optimized Variables to select variables to be optimized. You may also use Specify | Objective Function or Specify | Constraints to create a more complicated optimization. Then use Process | Optimize to run the optimization.

Menu item Tools | Explore Optimization displace information about the optimization specification in succinct form:

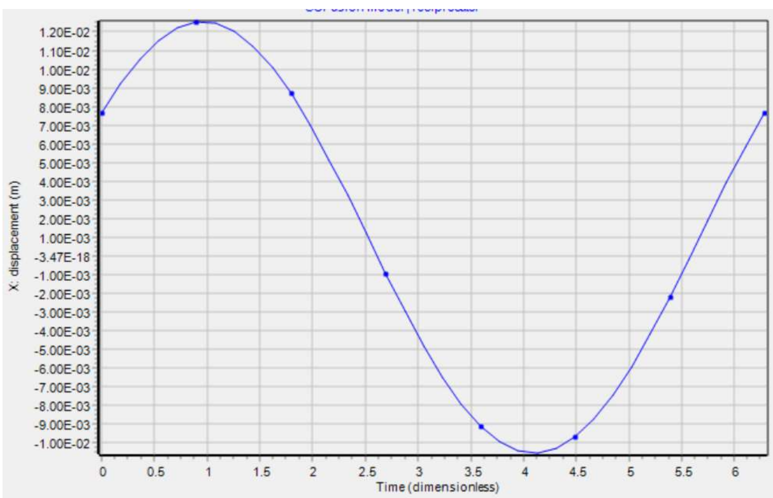
```
Objective Function
  Maximize Xamp

OPTIMIZED VARIABLES                 SUBJECT TO CONSTRAINTS

SCFusion model
  Freq
```

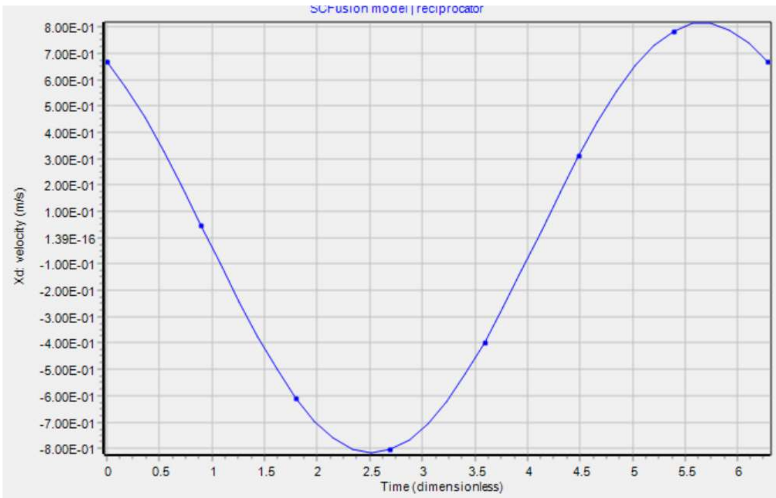
In this case there are no constraints.

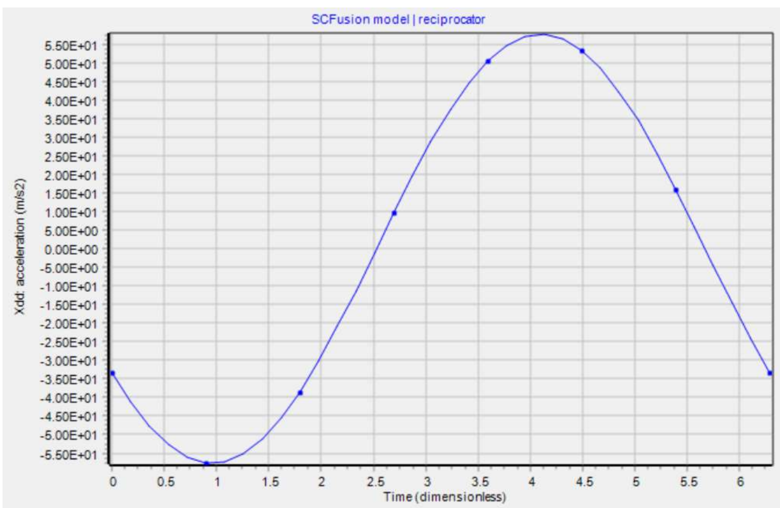
For more complicated optimizations the optimizer may fail to meet its target step change (convergence tolerance) due to numerical noise. This is normal. When the so-called Pseudo-Lagrangian step change is no longer decreasing and the optimized variables not changing much, it is reasonable to stop the solution as “close enough”. This amounts to a judgement call on your part. It is also possible for the optimizer to diverge as a result of poorly defined constraints or a particularly noisy model, in which case the optimized results are not to be trusted. The Sage User’s Guide (online help) talks more about working with the optimizer.

Interactive Plotting

Sage implements an interactive plotting routine for time-ring solution variables. For example, select the reciprocator model, click the right mouse button and select Plot Solution Grid in the popup context menu. You can then display the following plots.




SCFusion model | reciprocator
Xd: velocity (m/s)
Time (dimensionless)


SCFusion model | reciprocator
Xdd: acceleration (m/s2)
Time (dimensionless)